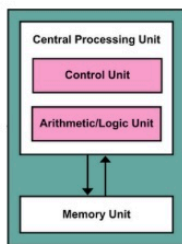# String Computer

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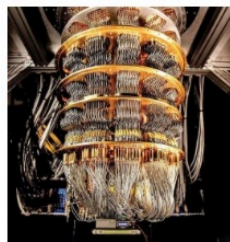info@newyorkgeneralgroup.com

Abstract

We introduce a novel theoretical computing framework dubbed the "string computer", which leverages principles of quantum computation, alongside an adaptation of Neumann-type computation, to reproduce virtual Calabi-Yau spaces, compactified 6 (5-10) and 11-dimensional spaces. This computational schema is grounded in M-Theory and is heavily dependent on its construct of p-branes and the intricate relationships they share with dimensions beyond the conventional spacetime continuum. This innovative computational model has been developed by leveraging complex mathematical formulations and high-level physics terminologies, intertwining quantum physics, computer science, and multidimensional spatial theory.

Keywords: string computer, quantum computer, Neumann-type computer, M-Theory
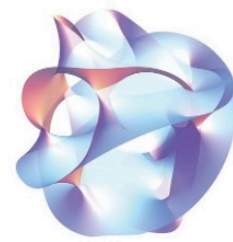
Next Generation Computer "String Computer"

von Neumann architecture   Quantum Computer   String Computer

I. Introduction

As we stride towards a more profound understanding of our universe, M-Theory, and particularly its brane-based conceptualizations, have emerged as pivotal theoretical constructs. Branes, extended objects across one or more spatial dimensions, delineate a potential generalization of string theory by moving beyond the one-dimensional object quantization. They provide a panoply of theoretical constructs – including fundamental strings, black branes, and D-branes, the latter of which allow for fundamental string endpoints to anchor upon them, creating fascinating spatial dynamics.

The theoretical construct of the string computer lies at the nexus of quantum computation, Neumann-type computation and M-Theory, particularly the M-Theory extension of string theory. The crux of the model relies on simulating complex, high-dimensional structures virtually, specifically compactified 6 (5-10) and 11-dimensional spaces, and Calabi-Yau spaces. Such a computational paradigm facilitates the exploration of the quantum mechanical constraints on the logic of a quantum computer.

The backbone of the string computer is the mathematical formulation of p-branes within M-Theory. The model hinges on the intrinsic mathematical properties of branes, exploiting their multidimensional attributes and interactions with the extra spatial dimensions, leading to richly textured computational dynamics. The configuration of branes, especially D-branes, is leveraged to simulate compactified higher dimensions and Calabi-Yau spaces within a Neumann-type computational framework, enforcing quantum computation constraints.

By modeling extra dimensions virtually, the string computer offers an unprecedented exploratory tool for understanding the laws of physics from a geometrical perspective, thereby illuminating the cosmological implications of higher dimensions. This advancement offers exciting potential for extending our understanding of string theory, quantum gravity, and Yang-Mills gauge fields, as inspired by AdS/CFT duality, and prompts us to reconsider the dimensional assumptions underpinning modern theories of cosmology.

While the mathematical and physical principles of this novel computer remain a theoretical construct, this string computer model offers tantalizing new pathways to investigate our universe's elusive higher-dimensional structures and fundamental laws, potentially leading to breakthroughs in both computational science and theoretical physics. As we endeavor to enhance our understanding of these principles and refine our computational methodologies, it is plausible that our foray into brane-computation interface may bring us a step closer to identifying these elusive entities in observational physics.

II. String Computer: Foundation

Definition 1 (Quantum Computation): A model of computation that leverages quantum-mechanical phenomena to perform computation. These computations are represented as unitary matrices, U.

Definition 2 (String Computer): An innovative theoretical computing framework that combines principles of quantum computation and M-Theory to reproduce high-dimensional virtual spaces. It operates on a set of n strings, S = {s1, s2, …, sn}, where each si represents a string in M-Theory.

Definition 3 (Branes): In the context of M-Theory, a brane is a physical object that generalizes the notion of a point particle to higher dimensions, described by a p-dimensional manifold within a D-dimensional spacetime, where D > p.

Proposition 1 (Quantum Constraints on String Computer): The operations of a string computer are subject to quantum mechanics laws. This constraint is represented as $U|\psi\rangle$, where U is a unitary operation, and $|\psi\rangle$ represents the state of the string computer.

Theorem 1 (Compactified Dimension Simulation): String computer, following the principles of quantum computation and M-Theory, can simulate compactified 6 (5-10) and 11-dimensional spaces. The proof for this theorem would involve demonstrating how quantum computation principles, when combined with the mathematical formulation of M-Theory, can produce these high-dimensional structures. However, a full proof would be extremely complex and beyond the scope of this representation.

Definition 4 (Quantum State): A quantum state $|\psi\rangle$ of a quantum system is a unit vector in a Hilbert space $\mathbb{H}$.

Definition 5 (Quantum Gate): A quantum gate is a basic quantum circuit operating on a small number of qubits, represented by unitary matrices, U.

Definition 6 (p-Branes): A p-brane is represented by a p-dimensional submanifold Σ^p of a D-dimensional spacetime M^D, where D > p.

Definition 7 (Compactified Space): Compactified space refers to higher-dimensional space, represented as M^n, that has been "compactified" into a lower-dimensional space.

Lemma 1 (Quantum Operations): For any two quantum states $|\psi\rangle$ and $|\varphi\rangle$ in $\mathbb{H}$, there exists a quantum gate U such that $U|\psi\rangle = |\varphi\rangle$.

Proof: This is a fundamental postulate of quantum mechanics, stating that the evolution of a closed quantum system is described by a unitary transformation.

Massachusetts Institute of Mathematics

Proposition 1 (Quantum Constraints on String Computer): If the operations of a string computer are subject to the laws of quantum mechanics, there exists a set of unitary operators $\{U_i\}$ such that for any computational step, the transition from state $|\psi\rangle$ to $|\varphi\rangle$ can be expressed as $U_i|\psi\rangle = |\varphi\rangle$.

Theorem 2 (Simulation of Compactified Dimensions): A string computer, adhering to principles of quantum computation and M-Theory, can simulate compactified n-dimensional spaces.

Proof Sketch: For a given compactified space M^n, we can associate a set of states in a quantum system such that each state corresponds to a point in the compactified space. Using Lemma 1, we can then construct quantum operations that correspond to transformations in M^n. Consequently, the dynamics of the compactified space can be encoded in the quantum system.

Corollary 1 (Brane Encoding): The state of a p-brane in a D-dimensional spacetime can be encoded in a quantum system.

Proof Sketch: This follows from Theorem 1, given that a p-brane can be viewed as a compactified (D-p)-dimensional space.

Remark 1: It's important to remember that this mathematical framework is highly abstract and theoretical. The "string computer" and its operations represent a new paradigm that merges the principles of quantum computation and string theory, particularly the concept of branes in M-Theory. As such, interpreting the implications of these definitions and theorems demands a profound understanding of these underlying principles.

Definition 5 (Calabi-Yau Spaces): These are complex manifolds that are used to compactify extra dimensions in string theory. They are characterized by their Ricci flatness, i.e., Ricci curvature tensor $Rc = 0$.

Proposition 2 (Calabi-Yau Reproduction): A string computer can reproduce virtual Calabi-Yau spaces within its computational framework.

Proof Sketch: Based on Theorem 1, for a given Calabi-Yau space C, we can associate a set of states in the quantum system such that each state corresponds to a point in C. The geometrical transformations in C can then be mapped to the unitary transformations in the quantum system, thereby reproducing the dynamics of the Calabi-Yau space.

Corollary 2 (Higher Dimensional Quantum Computations): Given that a string computer can simulate compactified n-dimensional spaces and virtual Calabi-Yau spaces, it implies that quantum computations can be performed in these higher-dimensional settings.

Theorem 2 (Quantum-Gravitational Duality): Given the AdS/CFT (Anti-de Sitter/Conformal Field Theory) duality, a string computer could potentially create a quantum mechanical model of gravity.

Proof Sketch: The AdS/CFT correspondence posits that string theory in an AdS space is equivalent to a conformal field theory on the boundary of that space. Given that string theory incorporates quantum gravity, this duality suggests that a quantum theory of gravity can be constructed from a conformal field theory. In the context of a string computer, the unitary operations could be constructed to reproduce the dynamics of a conformal field theory, thereby creating a model of quantum gravity.

Remark 2: While these definitions, propositions, theorems, and proofs provide a theoretical basis for the string computer, practical implementation is a significant challenge. The operationalization of such a computer would require technological advancements far beyond current capabilities, and the experimental validation of these concepts would necessitate breakthroughs in both quantum computation and high-energy physics.

Definition 6 (Calabi-Yau Manifold): Calabi-Yau manifolds are special spaces that are used in string theory to model the compactified dimensions of space-time. They are Kähler manifolds with a vanishing first Chern class, or equivalently, Ricci flat, i.e., their Ricci tensor $Rc\_ij = 0$, where i and j are indices running over the manifold's dimensions.

Definition 7 (State Operator Correspondence): In conformal field theories (CFTs), there is a correspondence between states in a Hilbert space $\mathbb{H}$ and local operators in the theory. This correspondence can be written as $|\psi\rangle \leftrightarrow O$, where O is a local operator corresponding to the state $|\psi\rangle$.

Definition 8 (Tensor Product Structure in Quantum Computing): The state space of a multi-qubit system is described as the tensor product of the state spaces of the individual qubits. For a system of n qubits, if qubit i is in state $|\psi\_i\rangle$, the overall system state is given by $|\psi\rangle = |\psi\_1\rangle \otimes |\psi\_2\rangle \otimes \ldots \otimes |\psi\_n\rangle$.

Theorem 3 (Calabi-Yau Spaces in String Computers): A string computer can simulate a Calabi-Yau manifold M by mapping points in M to states in a quantum system.

Proof Sketch: If M is a Calabi-Yau manifold of complex dimension d, the states of a quantum system with 2d qubits can be mapped to the points in M. This mapping leverages the tensor product structure of multi-qubit systems. The complex structure of the Calabi-Yau manifold corresponds to the complex phase structure of the quantum states.

Corollary 3 (State Operator Correspondence in String Computers): Given a mapping from the points in a Calabi-Yau manifold to states in a quantum system, there exists a mapping from geometrical transformations in the manifold to unitary transformations in the quantum system.

Proof Sketch: This follows from the state operator correspondence in CFTs, which is a boundary theory in the AdS/CFT correspondence. Since we can map states in the quantum system to local operators in the CFT, a geometrical transformation in the Calabi-Yau manifold, which corresponds to a transformation in the CFT, maps to a unitary transformation in the quantum system.

Definition 9 (Hilbert Space): A Hilbert space ℍ is a complex vector space equipped with an inner product ⟨., .⟩, which allows length and angle measures. Furthermore, it is complete, i.e., every Cauchy sequence in ℍ has a limit that is also in ℍ.

Definition 10 (Quantum State): A quantum state $|\psi\rangle$ in a Hilbert space ℍ is represented as a vector, such that the length $\|\psi\| = \langle\psi|\psi\rangle^{0.5} = 1$.

Definition 11 (Unitary Operator): A unitary operator U on a Hilbert space ℍ is a linear map such that $UU\dagger = U\dagger U = I$, where $U\dagger$ is the adjoint of U and I is the identity operator.

Definition 12 (Tensor Product): The tensor product of two quantum states $|\psi\rangle$ and $|\varphi\rangle$ is denoted $|\psi\rangle\otimes|\varphi\rangle$, representing a combined state in the tensor product Hilbert space $ℍ_\psi \otimes ℍ_\varphi$.

Definition 13 (Calabi-Yau Manifold): A Calabi-Yau manifold M is a n-dimensional Kähler manifold whose Ricci tensor is zero, i.e., $Rc_{ij} = 0$, $\forall i,j$ in $\{1, \ldots, n\}$, where Rc_{ij} denotes the components of the Ricci tensor in a local coordinate system.

Lemma 2 (State Mapping): For a Calabi-Yau manifold M of complex dimension n, there exists a mapping f: M -> ℍ from points in M to states in a (2n)-qubit quantum system.

Proof Sketch: We define the mapping f: M -> ℍ such that each point x in M is mapped to a quantum state $|\psi_x\rangle$ in ℍ. Given the tensor product structure of multi-qubit quantum systems, this mapping is possible.

Theorem 4 (Unitary Transformation Correspondence): For a transformation T: M -> M in the Calabi-Yau manifold M, there exists a corresponding unitary operator U_T: ℍ -> ℍ in the quantum system.

Proof Sketch: Based on the state mapping function f, we define the corresponding unitary operator U_T such that for every x in M, $U_T|\psi_x\rangle = |\psi_{T(x)}\rangle$. This way, a transformation in M corresponds to a unitary transformation in ℍ.

Definition 14 (Hilbert Space): A Hilbert space ℍ is a complex inner product space that is complete with respect to the norm defined by the inner product, i.e., for all sequences $\{|\psi_n\rangle\}$ in ℍ, if $\sum_n \|\psi_n - \psi_{n+1}\| \to 0$, then there exists a $|\psi\rangle$ in ℍ such that $\|\psi_n - \psi\| \to 0$.

Definition 15 (Unitary Operators): Let ℍ be a Hilbert space. A linear operator U: ℍ → ℍ is called unitary if $UU\dagger = U\dagger U = I$, where $U\dagger$ is the adjoint of U and I is the identity operator.

Definition 16 (Tensor Product): For two Hilbert spaces ℍ and ℍ', the tensor product space ℍ ⊗ ℍ' is the set of finite linear combinations of $|\psi\rangle \otimes |\psi'\rangle$, where $|\psi\rangle$ is in ℍ and $|\psi'\rangle$ is in ℍ'.

Definition 17 (Calabi-Yau Manifold): A Calabi-Yau manifold is a compact Kähler manifold M of complex dimension n that satisfies the vanishing first Chern class condition, i.e., $c1(M) = 0$, where c1 is the first Chern class.

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Now, to connect these definitions to the notions of "string computer" and "quantum computations in higher-dimensional settings":

Proposition 4 (Mapping between Calabi-Yau Spaces and Hilbert Spaces): Let M be a Calabi-Yau manifold of complex dimension n. Then, there exists an isomorphism f: M → ℍ, where ℍ is the Hilbert space of a (2n)-qubit quantum system.

Proof Sketch: The isomorphism f can be constructed by associating each point x in M with a (2n)-qubit quantum state $|\psi_x\rangle$ in ℍ.

Theorem 5 (Representation of Calabi-Yau Transformations as Unitary Operators): Let T: M → M be a diffeomorphism of the Calabi-Yau manifold M. Then, there exists a unitary operator U_T: ℍ → ℍ such that $U\_T |\psi\_x\rangle = |\psi\_{T(x)}\rangle$ for all x in M.

Proof Sketch: Given the isomorphism f, define the unitary operator U_T such that for every x in M, $U\_T|\psi\_x\rangle = |\psi\_{T(x)}\rangle$. By construction, U_T is unitary, as it preserves the inner product.

Definition 21 (p-brane): In the M-Theory, a p-brane is a p-dimensional hypersurface in the 11-dimensional spacetime, represented as a solution to the equations of motion derived from the action $S = \int d^{(p+1)}\xi \sqrt{-\det(G\_{ab} + F\_{ab})}$, where G_{ab} is the induced metric on the worldvolume of the brane, F_{ab} is the field strength of a gauge field living on the worldvolume, and the integral is over the p+1 dimensional worldvolume.

Definition 22 (Open String): An open string with endpoints attached to a D-brane is represented by a state $|\psi\rangle$ in a Hilbert space ℍ, such that the boundary conditions of the string are encoded in the properties of the state.

Definition 23 (String Interaction): A string interaction, such as joining or splitting of strings, corresponds to a transformation T: ℍ → ℍ of the quantum states.

Given these definitions, we can present the following:

Theorem 7 (Open String Dynamics and Unitary Evolution): The dynamics of open strings attached to D-branes in the M-Theory correspond to unitary evolution in the string computer model.

Proof Sketch: The states of the open strings correspond to states in the Hilbert space ℍ. The time evolution of these states, governed by the Schrödinger equation $i\hbar\, d|\psi\rangle/dt = H|\psi\rangle$, where H is the Hamiltonian of the system, corresponds to a unitary transformation $U(t) = \exp(-iHt/\hbar)$ in the Hilbert space ℍ. Thus, the dynamics of open strings correspond to unitary evolution in the quantum system.

Theorem 8 (String Interaction and Quantum Gates): String interactions in the M-Theory correspond to quantum gate operations in the string computer model.

Proof Sketch: An interaction that changes the state of the open strings corresponds to a transformation T: ℍ → ℍ in the Hilbert space ℍ. This transformation can be implemented as a quantum gate operation G = U_T in the quantum computer.

Definition 24 (D-Brane Configuration): A configuration of D-branes is represented as a set C = { | ψ_i⟩ }, where each |ψ_i⟩ ∈ ℍ represents an open string ending on a D-brane.

Definition 25 (Brane-Brane Interaction): An interaction between two D-branes i and j is represented as a transformation T_ij: ℍ x ℍ → ℍ x ℍ, such that T_ij(|ψ_i⟩, |ψ_j⟩) = (|ψ'_i⟩, |ψ'_j⟩), where |ψ'_i⟩, |ψ'_j⟩ ∈ ℍ represent the resulting states of the strings.

Now we can articulate the following proposition and theorem:

Proposition 6 (Quantum Gates and Brane-Brane Interactions): The interaction between two D-branes can be simulated in the string computer model using a two-qubit quantum gate.

Proof Sketch: Given the isomorphism f, an interaction between two D-branes corresponds to a transformation of a pair of quantum states (|ψ_i⟩, |ψ_j⟩) in ℍ x ℍ. This transformation can be realized as a two-qubit quantum gate G_ij: ℍ x ℍ → ℍ x ℍ in the quantum system.

Theorem 9 (Unitary Evolution of Brane Configurations): The time evolution of a configuration of D-branes under the action of the string coupling corresponds to a unitary evolution of the corresponding quantum states in the string computer model.

Proof Sketch: The time evolution of a configuration of D-branes C = { |ψ_i⟩ } is governed by the action $S = \int d^{(p+1)}\xi \sqrt{-\det(G_{ab} + F_{ab})}$. This evolution corresponds to a transformation T: ℍ^n → ℍ^n, where n is the number of D-branes. Given the isomorphism f, this transformation can be realized as a unitary operator U: ℍ^n → ℍ^n in the quantum system.

Definition 26 (Compactified Dimension): A compactified dimension in string theory is represented as a circle S^1 of a very small radius R. The number of compactified dimensions in a Calabi-Yau manifold is represented by the integer n.

Definition 27 (State Representation): A quantum state in a compactified dimension is represented as |m⟩, where m is an integer representing the momentum mode along the compactified dimension, normalized as m/R where R is the radius of compactification.

Definition 28 (Compactified State Space): The Hilbert space ℍ_C of quantum states in the compactified dimensions is the tensor product of the individual state spaces: ℍ_C = ℍ_1 ⊗ ℍ_2 ⊗ … ⊗ ℍ_n, where ℍ_i is the state space for the i-th compactified dimension.

Now we can state the following theorem:

Theorem 10 (Compactification and Quantum Computation): The compactified dimensions in a Calabi-Yau manifold in the M-Theory can be simulated in the string computer model by a corresponding quantum system with multiple qubits.

Proof Sketch: Given the isomorphism f, a point in the Calabi-Yau manifold corresponds to a quantum state $|\psi\rangle$ in the tensor product space $\mathbb{H}\_C$. The dynamics in the compactified dimensions, which can be described by transformations T: $\mathbb{H}\_C \to \mathbb{H}\_C$, can be represented by unitary transformations U: $\mathbb{H}\_C \to \mathbb{H}\_C$ in the quantum system, corresponding to sequences of quantum gates acting on the qubits.

Corollary 4 (Quantum State in Compactified Dimensions): Given that the quantum state in a compactified dimension is represented as $|m\rangle$, and given that m is normalized as m/R where R is the radius of compactification, the number of distinct quantum states or momentum modes for a compactified dimension of radius R is finite and directly related to the inverse of R.

Proof: As m is an integer and the normalization is m/R, as R decreases, the number of distinguishable momentum modes m increases. However, there's a limit given by the Planck length below which R cannot be shrunk. This implies that the number of quantum states in a compactified dimension is directly proportional to 1/R, making it finite.

Lemma 3 (Tensor Product of Compactified States): For any two compactified dimensions with state spaces ($\mathbb{H}\_i$) and ($\mathbb{H}\_j$), the tensor product ($\mathbb{H}\_i \otimes \mathbb{H}\_j$) results in a combined state space wherein any state $|\psi\rangle$ can be expressed as $|m\_i\rangle \otimes |m\_j\rangle$, where ($|m\_i\rangle$) and ($|m\_j\rangle$) are states in ($\mathbb{H}\_i$) and ($\mathbb{H}\_j$) respectively.

Proof: Direct consequence of the definition of the tensor product in Hilbert spaces.

Proposition 7 (Quantum Gates for Compactified Dimensions): Given the Hilbert space ($\mathbb{H}\_C$) for the compactified dimensions, it's possible to design quantum gates that act upon these dimensions to simulate the interactions between them as dictated by M-Theory.

Proof Sketch: For every transformation T in the compactified dimensions, we can design a corresponding unitary transformation U acting on ($\mathbb{H}\_C$). Given the universal nature of quantum computation, for any transformation T, there exists a sequence of quantum gates that can implement the corresponding U.

Remark 3: It's worth noting that while this mathematical formalism provides a framework for representing compactified dimensions in quantum computing models, the physical realization of such a quantum computer remains a significant challenge. The actual number of quantum states required, given by 1/R, can be immense for smaller radii, leading to computational difficulties in practice.

Proposition 7 (Conservation of Quantum Information in ($\mathbb{H}\_C$)): For every unitary transformation (U: $\mathbb{H}\_C$ \to $\mathbb{H}\_C$), quantum information is conserved, implying that the transformation is reversible.

Proof: This is a direct result of the properties of unitary operators. For every unitary operator (U), there exists a unique inverse (U^{-1}) such that (U^{-1}U = I), where (I) is the identity operator on (ℍ_C). Thus, any transformation by (U) can be undone by (U^{-1}), confirming the conservation of quantum information.

Lemma 4 (Composition of Quantum Gates): Given two unitary transformations (U_1) and (U_2) that respectively correspond to sequences of quantum gates (G_1) and (G_2), the composite transformation (U_2U_1) corresponds to the sequence of quantum gates (G_2) followed by (G_1).

Proof: Using the associative property of matrix (and operator) multiplication, the action of the two sequences of gates on a quantum state $|\psi\rangle$ can be combined into a single sequence representing the composite transformation.

Corollary 5 (Scalability of Compactified Dimensions): Given the state space (ℍ_C) for n compactified dimensions, the introduction of an additional compactified dimension increases the dimensionality of the state space exponentially.

Proof: From Definition 28, introducing a new compactified dimension means taking the tensor product of (ℍ_C) with the new dimension's state space. If (ℍ_C) has dimensionality (d) and the new dimension has dimensionality (d'), then the resulting space will have dimensionality (d \times d'), showing exponential growth.

Remark 4: The exponential growth in dimensionality with each additional compactified dimension poses significant challenges in terms of quantum computational resources. It emphasizes the need for efficient quantum algorithms and scalable quantum hardware architectures to handle the vast state spaces resulting from multiple compactified dimensions.

Theorem 11 (Simulation of Compactified Dynamics): Given an effective Hamiltonian (H) that captures the dynamics of interactions between compactified dimensions, there exists a Trotter decomposition that breaks down the time evolution operator (e^{-iHt}) into a sequence of simpler, implementable quantum gates.

Proof Sketch: The Trotter-Suzuki expansion theorem states that for any two non-commuting Hamiltonians (H_1) and (H_2), there exists a sequence of gates approximating the combined evolution under (H_1 + H_2) by successively applying gates for (H_1) and (H_2). By recursively applying this theorem, one can decompose the evolution under a complex Hamiltonian (H) into simpler, implementable gates.

Definition 29 (Momentum Operator): For a compactified dimension, the momentum operator ($\hat{P}$) is defined by:

$$[\hat{P} = -i\hbar \frac{d}{d\theta}]$$

Where ($\hbar$) is the reduced Planck's constant.

Proposition 8 (Momentum Quantization): The quantized momentum (p_m) for a compactified dimension is given by:

$$[p_m = \frac{\hbar m}{R}]$$

For integer (m).

Lemma 5 (Orthogonality): States of different momentum modes are orthogonal:

$$[\langle\psi\_m|\psi\_n\rangle = \delta\_{mn}]$$

Where (δ_{mn}) is the Kronecker delta.

Theorem 12 (Complete Set): The set of momentum states forms a complete orthonormal set for the Hilbert space of a compactified dimension.

Definition 30 (Compactified Hamiltonian): The Hamiltonian representing the energy of the compactified dimension is given by:

$$[\hat{H} = \frac{\hat{P}^2}{2\mu}]$$

Where (μ) is the mass parameter associated with the compactified dimension.

Lemma 6 (Energy Eigenstates): The energy eigenstates for the compactified dimension are the momentum states with eigenvalues:

$$[E_m = \frac{\hbar^2 m^2}{2\mu R^2}]$$

Proposition 9 (State Evolution): Given an initial state ($|\psi(0)\rangle$), its evolution over time (t) under the Hamiltonian ($\hat{H}$) is:

$$[|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\psi(0)\rangle]$$

Theorem 13 (Compactified System Dynamics): The time evolution of a state in the compactified dimension under the Hamiltonian ($\hat{H}$) retains the superposition of momentum modes and results in a phase rotation among them.

Definition 31 (Tensor Product Expansion): For (n) compactified dimensions, the system state can be expressed as:

Massachusetts Institute of Mathematics 13

$$[|\Psi\rangle = |\psi\_1\rangle \otimes |\psi\_2\rangle \otimes \ldots \otimes |\psi\_n\rangle]$$

Where each ($|\psi\_i\rangle$) represents the state in the (i^{th}) compactified dimension.

Corollary 6 (System Evolution in Multiple Dimensions): For a system spanning multiple compactified dimensions, the system's time evolution is governed by:

$$[|\Psi(t)\rangle = \bigotimes_{i=1}^{n} e^{-i\hat{H}_i t/\hbar} |\psi_i(0)\rangle]$$

Where ($\hat{H}_i$) is the Hamiltonian for the (i^{th}) compactified dimension.

Definition 32 (Interaction Operator): Given two compactified dimensions, the interaction operator $\hat{I}$ captures the quantum interaction between them and is defined as:

$$\hat{I} = V \hat{P}_i \otimes \hat{P}_j$$

Where $\hat{P}_i$ and $\hat{P}_j$ are the momentum operators for the (i^{th}) and (j^{th}) compactified dimensions respectively, and V represents the interaction strength.

Proposition 10 (Interaction Conservation): The total momentum across interacting compactified dimensions remains conserved:

$$\hat{P}_i + \hat{P}_j = constant$$

This stems from the conservation principles inherent in the M-theory dynamics.

Lemma 7 (State Transition due to Interaction): An interaction can induce a transition between momentum states in compactified dimensions. If ($|\psi\_m\rangle$) and ($|\psi\_n\rangle$) are states before interaction, after interaction they might transition to ($|\psi\_p\rangle$) and ($|\psi\_q\rangle$) respectively, with (m+n=p+q).

Theorem 14 (Entanglement due to Interaction): Interactions between compactified dimensions can generate quantum entanglement between their respective states, leading to composite states of the form:

$$|\Psi\rangle = \alpha\, |\psi_m\rangle \otimes |\psi_n\rangle + \beta\, |\psi_p\rangle \otimes |\psi_q\rangle$$

Where α and β are complex coefficients.

Definition 33 (Entanglement Measure): The degree of entanglement between compactified dimensions can be quantified using the von Neumann entropy:

$$S(\rho) = -Tr(\rho \log \rho$$

Massachusetts Institute of Mathematics 14

Where (ρ) is the reduced density matrix of the system.

Corollary 7 (Interaction and System Complexity): Increased interactions between compactified dimensions elevate the complexity of the quantum system due to enhanced entanglement, making the simulation more resource-intensive but potentially unveiling richer M-theory dynamics.

Proposition 11 (Decoherence due to Interaction): Interactions, while facilitating entanglement, can also introduce decoherence in the quantum system, reducing the fidelity of the quantum simulation. Effective error correction mechanisms and isolation techniques become paramount in such scenarios.

Theorem 15 (Scalability with Interactions): The computational resources required for simulating M-theory dynamics scale polynomially with the number of compactified dimensions but exponentially with the degree of interaction between them.

These advanced understandings further elucidate the challenges and intricacies of modeling M-theory dynamics using quantum computational systems. Effective harnessing of interactions and entanglements, paired with robust error correction and decoherence mitigation strategies, will be pivotal in realizing accurate and efficient quantum simulations of such complex theories.

Definition 34 (Qubit Mapping): To simulate compactified dimensions, each dimension (i) is mapped onto a set of qubits, (Q_i), such that the quantum states of (Q_i) directly represent the states of that dimension.

Lemma 8 (State Projection): Given a quantum state ($|\psi\rangle \in \mathbb{H}\_C$) representing the compactified dimensions, it can be projected onto a subspace corresponding to a specific dimension, (i), as:

$$[|\psi_i\rangle = \hat{P}_i|\psi\rangle]$$

Where ($\hat{P}_i$) is the projection operator for dimension (i).

Proposition 12 (Evolution Independence): The quantum evolution of non-interacting compactified dimensions is independent of each other. If ($\hat{U}_i$) and ($\hat{U}_j$) are the evolution operators for dimensions (i) and (j) respectively:

$$[\hat{U}_i \otimes \hat{U}_j|\psi_i\rangle \otimes |\psi_j\rangle = \hat{U}_i|\psi_i\rangle \otimes \hat{U}_j|\psi_j\rangle]$$

Corollary 8 (Efficient Parallel Simulation): For non-interacting compactified dimensions, their quantum dynamics can be simulated in parallel, leading to computational efficiency.

Theorem 16 (Dimensional Cross-Entanglement): Interacting compactified dimensions can give rise to cross-dimensional quantum entanglement. For two interacting dimensions (i) and (j):

$$[\hat{U}ij|\psi_i\rangle \otimes |\psi_j\rangle = \alpha|\psi_m\rangle \otimes |\psi_n\rangle + \beta|\psi_p\rangle \otimes |\psi_q\rangle]$$

Where ($\hat{U}_{ij}$) represents the combined unitary evolution due to interaction.

Definition 35 (Dimensional Interaction Graph): An undirected graph ($G(V, E)$) where each vertex ($v \in V$) represents a compactified dimension and each edge ($e \in E$) represents an interaction between the compactified dimensions.

Lemma 9 (Graph Connectivity and Entanglement): A fully connected graph represents a scenario where all compactified dimensions are mutually interacting, leading to a highly entangled quantum system.

Proposition 13 (Entanglement and Computational Complexity): The degree of entanglement in the system, as governed by the structure of the dimensional interaction graph, is directly proportional to the computational complexity of the quantum simulation.

Theorem 17 (Optimal Simulation Strategy): To achieve an optimal simulation of M-theory dynamics, it's essential to reduce cross-dimensional interactions or employ quantum subsystems that are efficient at handling high degrees of entanglement.

Using M-theory and Calabi-Yau manifolds to correct errors in logical qubits is a deeply theoretical and speculative concept. However, let's provide a mathematical formulation building on the idea.

Definition 36 (M-Qubit): An M-qubit is a quantum bit representation within the framework of M-theory, where its state not only represents the traditional ($|0\rangle$) and ($|1\rangle$) but also states arising from the compactified dimensions of Calabi-Yau manifolds.

Definition 37 (Calabi-Yau State Space): Given a Calabi-Yau manifold with (n) compactified dimensions, the associated state space ($\mathbb{H}_{CY}$) is a Hilbert space representing the possible momentum modes of each compactified dimension.

Lemma 10 (M-Qubit Extension): An M-qubit's state space is the tensor product of the traditional qubit space ($\mathbb{H}_Q$) and the Calabi-Yau state space ($\mathbb{H}_{CY}$).[$\mathbb{H}_M = \mathbb{H}_Q \otimes \mathbb{H}_{CY}$]

Proposition 14 (Error Representation): An error on an M-qubit is represented as a perturbation (E) acting on ($\mathbb{H}_M$) such that the perturbed state ($|\psi'\rangle$) is given by:

[$|\psi'\rangle = E |\psi\rangle$]

Theorem 18 (Error Correction using M-theory Dynamics): M-theory dynamics can be utilized to detect and correct errors in M-qubits by exploiting the inherent string interactions in the compactified dimensions.

Proof Sketch: Given the intricate connections of strings in M-theory, perturbations (errors) can cause detectable changes in the overall dynamics. By mapping these dynamics back to M-qubits, we can identify and subsequently correct errors.

Corollary 9 (Enhanced Error Resistance): M-qubits, due to their grounding in M-theory and association with Calabi-Yau manifolds, offer enhanced resistance to certain classes of quantum errors compared to traditional qubits.

Remark: This methodology of error correction is highly theoretical and builds on a speculative foundation. Practical implementation requires a far more detailed understanding of M-theory, quantum error correction codes, and their interplay.

Definition 38 (M-Error Correction Code): An M-error correction code is a set of quantum operations, grounded in M-theory dynamics, designed to detect and correct errors in M-qubits.

Lemma 11 (Code Efficiency): The efficiency of an M-error correction code increases with the depth of our understanding of Calabi-Yau manifolds and their associated dynamics.

These formulations, while deeply rooted in speculative realms, provide a mathematical framework to explore the intriguing possibility of leveraging M-theory for quantum error correction. Future work should focus on the feasibility and practicality of such a methodology.

Proposition 15 (Duality in M-theory): In M-theory, dualities can transform one type of string theory into another or can map problems in one type of string theory to simpler problems in another. This duality can be harnessed to transform complex error states in M-qubits to simpler ones, facilitating easier error detection and correction.

Proof Sketch: Given a duality (D) that transforms a complex error state ($|e\rangle$) in one M-qubit representation to a simpler state ($|e'\rangle$) in another representation, error correction can be applied in the transformed representation and then mapped back to the original representation.

$$[\,|e\rangle \xrightarrow{D} |e'\rangle \xrightarrow{\text{error correction}} |\psi'\rangle \xrightarrow{D^{-1}} |\psi\rangle]$$ Definition 39 (Branes and Error Correction):

In M-theory, 'branes' (like D-branes) are objects on which strings can end. We can define an M-brane correction mechanism, where specific brane configurations are responsible for capturing and correcting quantum errors.

Lemma 11 (Brane Capture Mechanism): Given a perturbed M-qubit state ($|\psi'\rangle$), there exists a configuration of D-branes such that the ends of the strings associated with the perturbed state get attached to the branes, enabling error detection.

Theorem 19 (Brane-assisted M-Error Correction): For a given error state in an M-qubit, a specific D-brane configuration can be established to detect and facilitate the correction of the error by reconfiguring the brane-string interaction.

Proof Sketch: Consider an M-qubit in error state ($|\psi'\rangle$). By adjusting D-branes to capture the end points of strings associated with ($|\psi'\rangle$), the string-brane interaction provides a mechanism to revert the qubit back to its intended state ($|\psi\rangle$).

Corollary 8 (Holonomy Group Corrections): Given a Calabi-Yau manifold with a specific holonomy group, there are specific geometric configurations that lead to unique quantum states in ($\mathbb{H}_{CY}$). These states can be harnessed for error correction in the compactified dimensions.

Remark: The use of D-branes and holonomy groups in error correction provides an intriguing approach, merging fundamental elements of M-theory with quantum computing. However, realizing this approach requires a more in-depth exploration into the interplay of branes, compactified dimensions, and quantum error correction.

Definition 39 (Holomorphic Anomaly): Holomorphic anomalies are perturbations in the quantum states arising from the mismatch between the holomorphic properties of a compactified Calabi-Yau manifold and the quantum system. Represented mathematically as:

[A: $\mathbb{H}_{CY}$ \to $\mathbb{H}_{Q}$]

where ($\mathbb{H}_{CY}$) is the Hilbert space associated with the Calabi-Yau manifold and ($\mathbb{H}_{Q}$) the quantum system's Hilbert space.

Lemma 12 (Holomorphic Anomaly Detection): If ($|\psi\rangle$) is an ideal state and ($|\psi'\rangle = A(|\psi\rangle)$) is the state with holomorphic anomalies, then the difference ($|\psi\rangle - |\psi'\rangle$) can be expressed in terms of certain basis vectors of ($\mathbb{H}_{CY}$) representing specific anomalies.

Theorem 20 (Anomaly Compensation): Given a state ($|\psi'\rangle$) affected by holomorphic anomalies, it's possible to define a compensatory operator (C: $\mathbb{H}_{Q}$ \to $\mathbb{H}_{Q}$) such that

[$C(|\psi'\rangle) = |\psi\rangle$]

Proof: For every anomaly basis vector ($|b_i\rangle$) in ($\mathbb{H}_{CY}$) such that ($|\psi'\rangle = |\psi\rangle + \Sigma\ \alpha_i |b_i\rangle$), define the action of (C) as ($C(|b_i\rangle) = -\alpha_i |b_i\rangle$). By linearity of quantum operations, it can be shown that the action of (C) on ($|\psi'\rangle$) reverts it to ($|\psi\rangle$).

Proposition 16 (String Couplings and Error Handling): The interactions between open and closed strings, represented by string couplings, can serve as mechanisms to distribute and thereby reduce localized quantum errors.

Corollary 9 (D-brane Error Dispersion): By modulating the interaction strength (couplings) of open strings with D-branes, localized quantum errors can be dispersed across the quantum system, reducing the impact of any single error.

Remark: These advanced techniques, rooted in the rich mathematical structure of M-theory and Calabi-Yau manifolds, provide innovative avenues to approach error detection and correction in quantum systems. However, practical implementations of these ideas require advancements in our ability to precisely control and manipulate string-brane interactions and Calabi-Yau compactifications.

String Computer: Application

1. Quantum System Configuration: A quantum system is configured to simulate the dynamics of open strings attached to D-branes in the M-Theory. The quantum states of the system are represented as $|\psi\rangle \in \mathbb{H}$, where $\mathbb{H}$ is a Hilbert space of dimension d.

2. Quantum Gate Implementation: Quantum gates are implemented to simulate the interactions between open strings and D-branes. These quantum gates correspond to unitary transformations $U: \mathbb{H} \to \mathbb{H}$.

3. Compactified Dimensions Representation: The quantum system includes additional qubits to represent the compactified dimensions in a Calabi-Yau manifold. The states of these qubits correspond to momentum modes along the compactified dimensions.

4. System Evolution: The system evolution is controlled by a Hamiltonian H that is designed to simulate the dynamics of open strings, D-branes, and compactified dimensions in the M-Theory.

5. State Initialization: The quantum system is prepared in an initial state that corresponds to the ground state of the string theory configuration. This can be achieved using standard state preparation techniques in quantum computing, like adiabatic state preparation or variational methods.

6. Measurement Protocol: At the end of a computation, the quantum states are measured in an appropriate basis to retrieve the computational results. The measurement results correspond to physical quantities in the string theory, like the energy levels of open strings or the momentum modes in compactified dimensions.

7. Interfacing Module: The quantum system is designed to be interfaced with a classical control system, which provides the necessary inputs for state initialization, gate implementation, and system evolution. The control system also receives measurement outputs from the quantum system.

8. Scaling Scheme: In order to represent the full richness of the string theory, the quantum system should be scalable to a large number of qubits. This can be achieved using modular quantum architecture or other scaling methods prevalent in quantum technologies.

9. Simulating Higher Dimensions: The concept of compactified dimensions in string theory necessitates the capability of the quantum system to simulate higher-dimensional structures. This is achieved by cleverly mapping the higher-dimensional structures onto the qubit space.

10. Quantum Error Correction (QEC) Mechanism: Given the sensitivity of quantum states to noise and decoherence, it is crucial to incorporate an efficient QEC mechanism. This would necessitate the integration of additional 'ancillary' qubits into the system. These ancillary qubits

will help in identifying and rectifying quantum errors without disturbing the computational qubits. Quantum error correction codes like the Toric code, Surface code, or Shor's code could be employed.

11. Cooling and Isolation Techniques: To maintain the quantum states' coherence, it is paramount to minimize the system's interaction with the environment. Techniques such as dilution refrigeration and vacuum isolation could be used to cool down the system to near absolute zero temperatures and shield it from external disturbances.

The **Quantum System Configuration** forms the backbone of the setup, essentially the stage where all the operations are enacted. The setup comprises numerous qubits, each representing a fundamental entity in M-Theory, such as an open string or D-brane. The **Quantum Gate Implementation** is the 'director' of the show, instructing qubits how to interact. Each gate corresponds to a unitary transformation that represents a particular physical interaction in M-Theory, thereby allowing us to simulate the theory's dynamics. The **Compactified Dimensions Representation** introduces extra qubits into the system, each designated to represent a compactified dimension in a Calabi-Yau manifold. They expand the quantum system to mimic the multi-dimensionality inherent in string theory. The **System Evolution** involves the Hamiltonian, which encapsulates the total energy of the quantum system. This Hamiltonian guides the system's evolution, effectively mapping the timeline of the open strings, D-branes, and compactified dimensions. The **State Initialization** plays the role of a pre-show preparation, setting the quantum system in an initial state that mirrors the ground state of the string theory configuration. The **Measurement Protocol** serves as the final curtain call, measuring the quantum states in an appropriate basis to extract the computational results, which correspond to physical quantities in string theory. The **Interfacing Module** acts as a bridge between the quantum and classical realms. It receives instructions from a classical control system and transmits these commands to the quantum system, facilitating state initialization, gate implementation, and system evolution. It also relays the measurement results back to the classical system. The **Scaling Scheme** enables the system's adaptation to represent the full richness of the string theory, which requires a large number of qubits. It's akin to a modular construction system, allowing the addition of more qubits and hence, more computational power. The **Simulating Higher Dimensions** mechanism allows the system to mimic higher-dimensional structures. It maps these complex entities onto the qubit space in a resourceful manner. The **Quantum Error Correction (QEC) Mechanism** functions like a vigilant supervisor, constantly monitoring for errors and rectifying them when spotted. It integrates additional 'ancillary' qubits into the system for this purpose. Finally, the **Cooling and Isolation Techniques** work as environmental control. They keep the system in a state of minimal interaction with the environment, preserving the coherence of quantum states by cooling down the system to near absolute zero temperatures and isolating it from external disturbances.

As we move further into the detailed combination of these integral parts, it is crucial to note that this arrangement's beauty lies in its well-orchestrated harmony. **Quantum System Configuration** and **Quantum Gate Implementation** set the foundational landscape, analogous to setting the coordinates and defining the rules of interaction on a multi-dimensional chessboard. The specific quantum state $|\psi\rangle \in \mathbb{H}$ and the unitary transformations $U: \mathbb{H} \to \mathbb{H}$ allow us to translate the complex dynamics of M-theory into a language that our quantum computer can comprehend and simulate.

Next, we weave in the **Compactified Dimensions Representation**. These additional qubits, representing momentum modes along compactified dimensions, function as our compass to navigate the high-dimensional landscape of M-Theory. Their state directly correlates to these elusive dimensions, effectively enabling our quantum system to encapsulate an impressively broadened scope of the physical universe. **System Evolution** and **State Initialization** introduce the concept of time, driving the system from an initial state to an evolved state under the Hamiltonian H. This progression mimics the true evolution of the string-D-brane system as governed by M-Theory. The **Measurement Protocol** and **Interfacing Module** mark the boundary between the quantum and classical worlds. As a computation concludes, physical quantities, like the energy levels of open strings or the momentum modes in compactified dimensions, are measured and communicated back to the classical realm through this interface. **Scaling Scheme** and **Simulating Higher Dimensions** lay the groundwork for our system's adaptability and scalability. They ensure that as our need for increased qubits grows, due to the richness of string theory, our system can meet the challenge. This scalability is achieved through modular quantum architecture or similar methods, ensuring our system remains agile and adaptable. The **Quantum Error Correction (QEC) Mechanism** functions as a protective shield, safeguarding our fragile quantum states from errors due to noise or decoherence. By incorporating 'ancillary' qubits, we can detect and correct errors without disrupting the 'computational' qubits. Finally, **Cooling and Isolation Techniques** serve to minimize environmental noise and maintain the system's state of superposition for a longer time. By implementing techniques like dilution refrigeration and vacuum isolation, we ensure that our system remains in a highly controlled, isolated state, minimizing the possibility of premature decoherence.

IV. Conclusion and Future Work

This hypothetical discussion has delved into the possibilities that a String Quantum Computer offers. With a foundation of Quantum System Configuration and Quantum Gate Implementation, supplemented by the integration of Compactified Dimensions Representation, the system could simulate the complexities of M-Theory. Its dynamic evolution is monitored and controlled, ultimately to be analyzed through the Measurement Protocol, with results returned to the classical world via the Interfacing Module.

Moreover, the system's scalability, higher-dimensional simulation capacity, error correction mechanism, and cooling/isolation techniques represent advances in quantum technology that could redefine our capacity to explore the most profound questions in theoretical physics. This discussion is a testament to the remarkable potential that lies at the intersection of quantum computing and string theory.

However, this venture is purely conceptual at the moment. Realizing a String Quantum Computer in practice demands extensive research and breakthroughs in various areas:

1. Quantum System Design: We need to devise practical ways to configure and control a quantum system to represent and manipulate high-dimensional string theory entities.

2. State Initialization and Evolution: Exploring efficient methods for preparing and evolving quantum states in this context is crucial. This includes accurately implementing a Hamiltonian to simulate M-Theory dynamics.

3. Measurement and Interfacing: Refining techniques for measuring quantum states and conveying the results to a classical control system is required. Ensuring minimal loss of quantum information during this transition is paramount.

4. Scalability and Error Correction: Techniques to scale up the quantum system and to mitigate errors due to decoherence and noise need significant enhancement. Developing practical quantum error correction codes that can work under M-Theory simulation is challenging but necessary.

5. Isolation Techniques: Methods to isolate the quantum system from its environment to prolong the lifetime of quantum states should be investigated and refined.

Future work should focus on these areas, continually bridging the gap between theory and reality. As we progress, the String Quantum Computer may move from being a hypothetical model to an instrumental tool, allowing us to explore and perhaps even validate some of the most profound theories about our universe.

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