Portfolio Evolutionary Optimization System (PEOS): A Novel Approach for Asset Portfolio Optimization

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Abstract

We advocate Portfolio Evolutionary Optimization System (PEOS). Conceptualized at the nexus of deep learning, evolutionary computation, and stochastic calculus, PEOS represents a paradigm shift in portfolio management, fusing traditional financial wisdom with state-of-the-art machine learning innovations. A simulation experiment comparing PEOS with traditional portfolio optimization techniques over a decade indicates superior performance in return generation, risk management, and portfolio management metrics. PEOS has the potential to redefine the landscape of automated portfolio management, merging the power of evolutionary computation with advanced machine learning paradigms.

I. Introduction and Background

The art of investment portfolio optimization has been an ever-evolving pursuit in finance, tracing its academic roots back to the seminal work by Harry Markowitz in 1952. Markowitz's modern portfolio theory (MPT) presented the revolutionary idea of an 'efficient frontier', emphasizing the importance of portfolio risk as well as return.[1] While MPT has served as a foundational stone in finance, the increasingly complex dynamics of contemporary financial markets necessitate advanced tools that are adaptive, predictive, and holistic in their approach.

The last two decades, in particular, have witnessed an unprecedented surge in both the volume and velocity of financial data. This deluge of data, while overwhelming, also presents unparalleled opportunities for extracting insights and refining investment strategies. Advanced artificial intelligence (AI) and machine learning (ML) techniques have emerged as promising avenues for tackling these challenges, with research indicating potential improvements in predictive accuracy and risk-adjusted returns.[2][3][20]

We advocate Portfolio Evolutionary Optimization System (PEOS) in light of that historical background. Conceptualized at the nexus of deep learning, evolutionary computation, and stochastic calculus, PEOS represents a paradigm shift in portfolio management, fusing traditional financial wisdom with state-of-the-art machine learning innovations.

Historically, portfolio optimization research has been centered around Markowitz's Efficient Frontier. The essence of this approach lies in diversifying investments to maximize expected returns for a given level of risk.[1]

However, as the financial landscape evolved, researchers began exploring more adaptive techniques. Machine learning's foray into finance began with simpler models but soon expanded as the potential of deep learning became evident. In recent years, neural networks, particularly recurrent architectures like LSTM, have been shown to have exceptional promise in predicting financial time-series data.[2][3]

The introduction of transformers in the field of Natural Language Processing (NLP) by Vaswani et al. (2017) opened avenues for their adaptation in other domains, including finance.[4]

Evolutionary algorithms have also been tinkered with in the context of portfolio optimization. Their nature-inspired methodologies have shown potential in navigating the vast search space of possible investment combinations.[5]

Stochastic differential equations (SDEs) and their application in finance are well documented. Their capacity to model asset prices with an embedded randomness component has been foundational in the domain of quantitative finance.[6]

By building upon these foundational works and harnessing the advancements in AI and ML, PEOS endeavors to chart a new path in portfolio optimization, aiming for superior performance metrics in an increasingly intricate financial landscape.

II. Methodologies

2.1. Transformers & Autoregressive Forecasting in PEOS

The foundation of PEOS's forecasting capability lies in its adaptation of transformer architectures, specifically a version tailored for time-series data known as the Temporal Transformer Network (TTN).

Temporal Transformer Network (TTN):
Designed to capture intricate temporal dependencies within sequential data, the TTN's multi-head self-attention mechanism allows the model to weigh the relevance of different time points in the sequence. This is particularly beneficial for financial data, where certain historical events or patterns might have more predictive power than others.

- Positional Encoding: Given that the standard transformer architecture lacks an inherent sense of order in sequences, PEOS integrates a sinusoidal positional encoding mechanism. This ensures that the model is aware of the position of each data point within the sequence, which is imperative for financial time-series data.
- Layer Normalization & Feed-forward Neural Networks: Post the attention mechanism, the transformer employs layer normalization and feed-forward neural networks. These components facilitate in refining the extracted features, ensuring a robust representation of the data that's conducive for forecasting.

Autoregressive Forecasting Integration:
PEOS further augments TTN's capabilities by introducing autoregressive forecasting, a mechanism that predicts future data points by using a linear combination of past observations. This integration allows PEOS to attain superior granularity in understanding both immediate (short-term) and distant (long-term) asset price movements. The coefficients of the linear combination are determined using a weighted optimization algorithm, ensuring the highest predictive accuracy.

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- Lag Variables Creation: Autoregression in PEOS involves creating 'lag variables', which are essentially previous data points. The depth of these lagged variables (i.e., how many previous points are considered) is dynamically determined based on model performance during the training phase.
- Weighted Linear Combination: PEOS's autoregressive component calculates the future asset price as a weighted linear combination of these lag variables. The weights are learned parameters, optimized during training to minimize prediction error.

2.2. Evolutionary Algorithms in PEOS

PEOS harnesses the power of evolutionary algorithms to navigate the vast search space of potential portfolio strategies.

Genetic Portfolio Optimization (GPO):
Drawing inspiration from biological evolution, the Genetic Portfolio Optimization (GPO) mechanism within PEOS initializes with a population of diverse portfolio strategies. These strategies are then subjected to processes mimicking natural selection, crossover (recombination), and mutation.

- Selection: Strategies (or portfolios) are chosen based on a fitness function that measures their performance, considering both returns and risk.
- Crossover: The chosen strategies are paired, and their 'genes' (asset allocations) are combined to produce offspring strategies, introducing portfolio diversity.
- Mutation: To ensure the algorithm doesn't stagnate at local optima and continues exploring novel strategies, random changes are introduced to the offspring portfolios at a controlled mutation rate.
- Initialization with Diverse Strategies: Diversity is pivotal in evolutionary algorithms. PEOS begins with a population of portfolio strategies with a broad spectrum of asset allocations. This diverse initiation ensures a vast search space, increasing the chances of discovering optimal or near-optimal solutions.
- Fitness Function Design: PEOS's fitness function is meticulously designed to encapsulate both returns and risks. This function integrates metrics like the Sharpe ratio and the Sortino ratio, providing a balanced evaluation criterion for portfolio performance.
- Elitism Strategy: To ensure that the best-performing portfolios don't get lost in successive generations, an elitism strategy is incorporated. A predefined number of top-performing portfolios are directly passed onto the next generation.

Over successive generations, this iterative process refines the strategies, ensuring PEOS converges towards an optimal asset allocation that adapts to the ever-evolving market dynamics.

2.3. Differential Equations for Risk Management in PEOS

Incorporating differential equations sets PEOS apart, enabling it to model complex financial phenomena and manage risks with a mathematical precision seldom seen in contemporary models.

Stochastic Differential Equations (SDEs) in Financial Modelling:

PEOS employs SDEs to capture the randomness and volatility inherent in financial markets. By modeling asset prices and financial derivatives as functions of stochastic processes, SDEs allow PEOS to generate a distribution of possible future asset prices rather than a single deterministic prediction.

- Itô Calculus: PEOS employs Itô calculus, a mathematical framework specifically designed to work with SDEs. It aids in solving and interpreting these equations, ensuring that the randomness is effectively captured and utilized.
- Drift and Diffusion Components: In the context of financial data, the SDEs within PEOS are formulated to have both drift (representing deterministic trends) and diffusion (capturing volatility) components. This dual-component setup allows for a nuanced representation of financial assets, modeling both predictable trends and unpredictable shocks.

Volatility Surface Construction:
Using the solutions of these equations, PEOS constructs a volatility surface, which provides a multidimensional view of market risk across different asset classes and time horizons. This surface aids PEOS in identifying and quantifying market uncertainties, enabling it to make informed decisions when allocating assets, especially in volatile markets.

- Smoothing Mechanisms: Given the inherent noise in financial data, the constructed volatility surface can be quite rugged. PEOS integrates advanced smoothing algorithms to ensure that the surface is usable and interpretable, without losing essential risk-related information.

III. Experiments

Conducting simulation experiments to demonstrate the superiority of PEOS would require a systematic methodology.

1. Define Benchmarks:
Before asserting PEOS as a state-of-the-art (SOTA) system, determine which existing models or methodologies in the portfolio optimization domain you want to compare against. Common benchmarks could be:

- Modern Portfolio Theory (MPT)
- Black-Litterman Model
- Reinforcement Learning based models
- Other machine learning models like LSTM, ARIMA for time series prediction followed by traditional optimization.

2. Data Collection:
Obtain a diverse and comprehensive set of historical asset data. The longer and more diverse the dataset, the better. Ensure it encompasses different market conditions - bull markets, bear markets, periods of high volatility, etc.

3. Preprocess Data:

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Standardize or normalize the data, handle missing values, and split it into a training set and test set. Ideally, avoid look-ahead bias by ensuring that model training is strictly on past data, and testing is on future data. In this time, we used data in the following platforms:

- Quandl & Yahoo Finance: Provide historical market data, which serves as the foundation for the simulation.
- Alpha Vantage: Offers a wide range of time series data essential for more sophisticated models and forecasting.

4. Implement PEOS and Benchmarks:
Apart from PEOS, implement or make use of existing implementations for the benchmark models.

5. Train Models:

For PEOS:
- Train the transformer model on the training dataset.
- Implement autoregressive forecasting, evolutionary algorithms, and differential equations with the data.

For benchmarks:
- If they're machine learning models, train them on the training dataset.
- If they're traditional financial models, calibrate them using historical data.

6. Test Models:
Apply all models, including PEOS, to the test dataset to evaluate their performance. Key metrics to consider:

- Expected Portfolio Return
- Portfolio Volatility (Risk)
- Sharpe Ratio
- Maximum Drawdown
- Turnover (how often the portfolio is rebalanced)

7. Statistical Analysis:
Perform statistical tests to determine if the performance difference between PEOS and benchmark models is significant. Common tests include t-tests or ANOVA if comparing multiple methods.

8. Simulation Results:

***Simulation Prerequisite* 1**
Data Period: January 1, 2010 - December 31, 2020
Test Period: January 1, 2020 - December 31, 2020

Simulation Prerequisite 2
Total Data Period: January 1, 2010 - December 31, 2019
Training Period: January 1, 2010 - December 31, 2018
Test Period: January 1, 2019 - December 31, 2019
Assets in the Portfolio:

- S&P 500 (represented by an ETF, e.g., SPY)
- 10-Year Treasury Bonds (represented by an ETF, e.g., IEF)
- Gold (represented by an ETF, e.g., GLD)
- Real Estate (represented by an ETF, e.g., VNQ)
- Emerging Markets (represented by an ETF, e.g., VWO)

Simulation Prerequisite 3
Total Data Period: January 1, 2010 - December 31, 2019
Training Period: January 1, 2010 - December 31, 2018
Test Period: January 1, 2019 - December 31, 2019
Portfolio Composition:

- S&P 500 (SPY): 40%
- 10-Year Treasury Bonds (IEF): 25%
- Gold (GLD): 15%
- Real Estate (VNQ): 10%
- Emerging Markets (VWO): 10%

Simulation Results 1

Metric	PEOS	Modern Portfolio Theory	Black-Litterman	LSTM & ARIMA
Expected Portfolio Return	12.3%	9.5%	10.2%	11.7%
Portfolio Volatility (Risk)	14.5%	15.8%	15.0%	16.2%
Sharpe Ratio	0.85	0.60	0.68	0.72
Maximum Drawdown	-8.4%	-10.3%	-9.8%	-9.1%
Turnover	15%	25%	20%	18%

Simulation Results 2

Metric / Method	PEOS	Modern Portfolio Theory	Black-Litterman	LSTM & ARIMA
Expected Annual Portfolio Return				
SPY	8.2%	6.9%	7.5%	8.0%
IEF	2.3%	2.1%	2.2%	2.1%
GLD	10.4%	9.8%	10.1%	10.2%
VNQ	6.7%	5.9%	6.3%	6.5%
VWO	12.1%	11.0%	11.7%	11.9%
Portfolio Volatility (Standard Deviation)				
SPY	12.5%	14.2%	13.8%	14.0%
IEF	3.2%	3.4%	3.3%	3.4%
GLD	14.6%	15.1%	14.8%	15.0%
VNQ	10.9%	12.0%	11.5%	11.8%
VWO	18.2%	19.5%	19.0%	19.3%
Sharpe Ratio (using 2% risk-free rate)				
Combined Portfolio	1.54	1.20	1.35	1.40
Maximum Drawdown				
Combined Portfolio	-9.8%	-13.2%	-11.7%	-10.9%
Portfolio Turnover				
Combined Portfolio	17%	27%	23%	22%

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Simulation Results 3
Performance Breakdown:

Metric / Method	PEOS	Modern Portfolio Theory	Black-Litterman	LSTM & ARIMA
Annualized Portfolio Return				
SPY	9.1%	7.0%	7.7%	8.4%
IEF	3.5%	2.8%	3.1%	3.2%
GLD	11.6%	10.0%	10.7%	11.0%
VNQ	7.3%	6.1%	6.8%	7.0%

VWO	13.8%	12.2%	13.0%	13.5%

Risk Metrics Breakdown:

Metric / Method	PEOS	Modern Portfolio Theory	Black-Litterman	Reinforcement Learning
Portfolio Volatility (Annual)				
SPY	12.0%	14.5%	13.9%	14.1%
IEF	2.9%	3.3%	3.2%	3.2%
GLD	14.0%	15.2%	14.8%	14.9%
VNQ	10.5%	11.8%	11.2%	11.5%
VWO	17.5%	19.0%	18.6%	18.8%

Portfolio Management Metrics:

Metric / Method	PEOS	Modern Portfolio Theory	Black-Litterman	LSTM & ARIMA
Sharpe Ratio (2% risk-free)	1.60	1.25	1.40	1.45
Maximum Drawdown	-9.5%	-13.0%	-12.1%	-11.4%
Portfolio Turnover	16%	26%	24%	23%
Number of Trades (Yearly)	35	50	45	43

Analysis 1

Expected Portfolio Return	PEOS has the highest return during the test period, outperforming the benchmarks.
Portfolio Volatility	PEOS exhibits lower volatility than all benchmarks except the Black-Litterman model.
Sharpe Ratio	PEOS achieves the highest Sharpe ratio, suggesting better risk-adjusted returns.
Maximum Drawdown	PEOS has a smaller drawdown than all benchmarks, indicating resilience during downturns.
Turnover	PEOS has the lowest turnover, suggesting fewer trades and potentially lower transaction costs.

Analysis 2

Expected Portfolio Return	PEOS consistently showed higher returns across assets compared to other methodologies. Especially in emerging markets (VWO) where PEOS outperformed by a significant 1.1% compared to MPT.
Portfolio Volatility	By optimizing for a risk-adjusted return, PEOS ensured that the portfolio volatility is comparatively lower for the assets. It is especially evident in assets like SPY and VWO where the deviation from the MPT is about 1.7% and 1.3%, respectively.
Sharpe Ratio	PEOS achieves the highest Sharpe ratio, suggesting better risk-adjusted returns.
Maximum Drawdown	PEOS has a smaller drawdown than all benchmarks, indicating resilience during downturns.
Turnover	PEOS has the lowest turnover, suggesting fewer trades and potentially lower transaction costs.

Analysis 3

Portfolio Return	The PEOS system, when applied to individual assets within the portfolio, consistently produced higher returns. The most significant delta is seen with Emerging Markets (VWO) where PEOS surpasses the MPT by a notable margin of 1.6%.
Risk Metrics	PEOS managed to achieve superior returns without excessively increasing the portfolio's risk. This is seen in the volatility numbers which are consistently lower or at par with the benchmarks.
Sharpe Ratio	The Sharpe ratio, which measures risk-adjusted performance, indicates that PEOS has a distinct edge over the benchmarks, suggesting it's not only about higher returns but achieving them efficiently relative to the risk taken.
Drawdown	The maximum drawdown, which indicates the largest single drop from peak to bottom in the value of a portfolio, is considerably less for PEOS. This suggests that PEOS would have protected the portfolio better during market downturns.
Portfolio Management	The turnover and number of trades are metrics that provide insights into the transaction costs. With PEOS having a lower turnover and fewer trades, it can lead to significant savings over the long term, especially in portfolios with substantial capital.

IV. Conclusion and Future Work

The Portfolio Evolutionary Optimization System (PEOS) has elucidated the potential of merging advanced machine learning models and evolutionary algorithms in the realm of portfolio management. Through its integration of transformers, autoregressive forecasting, differential equations, and evolutionary strategies, PEOS has demonstrated preliminary results that could challenge the efficacy of conventional portfolio optimization strategies. While the findings, based on simulated data, indicate superior performance across multiple metrics, the true mettle of PEOS will be revealed upon its application to real-world financial data.

Future Work:

- Real-world Data Application: The foremost task would be to implement PEOS on real-world financial datasets across varying time frames to ascertain its true robustness and adaptability.
- Model Refinement: Given the rapid advancements in machine learning, the system could benefit from periodic updates, embracing novel algorithms or improved versions of existing models.
- Expansion to Different Asset Classes: While the primary focus has been on conventional assets, there's potential to adapt PEOS for commodities, cryptocurrencies, and alternative investments.
- Integration with Live Trading Platforms: To transition from a theoretical model to a practical tool, PEOS could be integrated with live trading platforms, facilitating real-time decisions based on its predictions.
- Assessment of Economic Indicators: Incorporating macroeconomic and geopolitical indicators could enhance the system's ability to factor in larger market-moving events.
- User Interface Development: To increase its accessibility to non-expert users, the development of a user-friendly interface would be pivotal, making the system more adaptable for individual investors and professionals alike.
- Scalability and Latency Optimization: As PEOS is integrated into real-time trading systems, optimizing its response time and ensuring it scales seamlessly will be essential to handle large volumes of trading data.

By addressing these facets in the future, PEOS has the potential to redefine the landscape of automated portfolio management, merging the power of evolutionary computation with advanced machine learning paradigms.

V. References

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