# YuM-Theory: Hypothesis that elementary particles are 3-dimensional voxels

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## Abstract

Elmentary particles were once thought of as 0-dimensional points, then as 1-dimensional strings, and nowadays as 2-dimensional membranes. We advocate YuM-Theory (comes from the point that it is a development of M-theory and from its proponent's name, Yu Murakami.) that elementary particles are 3-dimensional voxels and that the universe is a set of voxels. Furthermore, we have considered YuM-Theory from the quantum mechanics, quantum field theory, and string theory. We then organized our theory by category theory. It contains Kan extensions, limit and colimit formulae, preserving extensions, pointwise Kan extensions, density and formal category theory. Finally, we have described a possible future course of action to prove our theory.

**Keywords:** elmentary particle, YuM-Theory, quantum mechanics, quantum field theory, string theory, category theory

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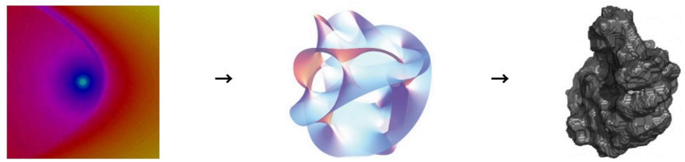

**Point Particle (0D)** **String or Membrane (1〜2D)** **Voxel (3D)**

# I. Introduction

Elmentary particles were once thought of as 0-dimensional points, then as 1-dimensional strings, and nowadays as 2-dimensional membranes.[16][17] We advocate YuM-Theory (comes from the point that it is a development of M-theory and from its proponent's name, Yu Murakami.) that elementary particles are 3-dimensional voxels and that the universe is a set of voxels.

Let us first dissect our understanding of particles as zero to two-dimensional entities. In the framework of quantum field theory, particles are indeed treated as 0-dimensional point-like entities, which is an abstracted interpretation that does a surprisingly good job of predicting experimental results. This theory forms the backbone of the incredibly successful Standard Model of particle physics.[1]

The leap to considering particles as 1-dimensional strings emerged from string theory, an ambitious endeavor to reconcile quantum mechanics with general relativity - the two major but incompatible pillars of modern physics. String theory envisions particles not as 0-dimensional points but rather as tiny, vibrating, 1-dimensional strings. Each mode of vibration of a string corresponds to a different particle. This theory, while mathematically elegant, is currently unproven due to the extreme energy scales necessary to test it directly.[1]

M-theory, a potential "theory of everything", is an extension of string theory and introduces higher-dimensional objects called p-branes, where the "p" denotes the number of dimensions. In this theory, 2-branes or membranes have played a pivotal role.[2]

But what of 3-dimensional "voxels"? This is a step beyond our current understanding and speculative at best. It would imply that elementary particles, rather than being points, strings, or membranes, are instead 3-dimensional volumetric elements. This would require a paradigm-shifting reworking of existing theories.

To formulate a rigorous mathematical proof, one would need to first develop a self-consistent mathematical framework that includes 3-dimensional voxel particles. This would likely involve extending the principles of quantum mechanics and quantum field theory into this new context, dealing with likely complications such as the question of how to define and compute quantum mechanical amplitudes for processes involving these 3D voxel particles.

After developing such a theory, it would need to make distinct predictions that can be tested against experimental results. This is a crucial point, as any physical theory must be able to predict the outcomes of experiments to be deemed accurate. If this voxel theory can predict known particle behavior and potentially solve unresolved issues, such as the nature of dark matter or the unification of gravity with the other fundamental forces, it could stand as a robust and viable theory.

However, as of now, there's no developed 3-dimensional voxel particle theory, and there's no experimental evidence suggesting particles have such a 3-dimensional structure.





If we consider the implication of viewing the universe as a set of 3-dimensional "voxels", we venture into the realms of digital physics and the concept of a discrete spacetime. Some scientists have proposed theories wherein spacetime itself is quantized, much like energy levels in quantum mechanics.

One such theory is the theory of Quantum Loop Gravity (QLG), which implies that spacetime might be composed of finite, discrete quantities, like "atoms" of spacetime. This theory predicts that the fabric of the universe itself is pixelated or voxelated in a sense, akin to how a seemingly smooth digital image is actually made up of finite, discrete pixels when magnified.[7]

However, QLG primarily deals with gravity, which is a different concept from considering particles themselves as being 3-dimensional voxels. While QLG proposes a discrete structure to spacetime[7], it doesn't provide an interpretation of particles as 3-dimensional entities.

To theoretically substantiate the proposition of 3D voxel particles, we must comprehend the potential consequences on fundamental principles of physics. For instance, how does a voxel-based perspective impact the Heisenberg Uncertainty Principle or the Pauli Exclusion Principle? How are Feynman diagrams and path integrals, key tools in quantum field theory, affected by the voxelation of particles? What does a voxel-based perspective imply for the nature of fundamental forces, such as the electromagnetic or the strong nuclear force?

Such a radical shift from accepted models of quantum mechanics, quantum field theory, and general relativity requires a solid mathematical foundation and theoretical framework, which, as far as our current understanding extends, does not exist. This should not discourage us, though, as breakthroughs in theoretical physics often come from daring hypotheses.

On the experimental side, testing this voxel concept would likely require much higher energy scales than current accelerators can achieve, given that the scales involved would be many orders of magnitude smaller than even the smallest particles we've observed so far. Experimental validation could come indirectly through cosmological observations, similar to how evidence for inflation comes from the cosmic microwave background radiation.

Delving deeper into the idea of 3-dimensional voxel particles, let's consider the possible implications for key areas of quantum mechanics and quantum field theory.

From a quantum mechanical perspective, envisioning particles as voxels introduces a conundrum. The cornerstone of quantum mechanics is wave-particle duality, where particles like electrons and photons exhibit both wave-like and particle-like properties. How can this be reconciled with a voxel-like particle, which presumably has fixed boundaries and a definite volume?

For instance, the phenomenon of quantum tunneling, where particles can pass through barriers that would be insurmountable under classical physics, could be profoundly affected. The wave-like nature of particles allows for the probabilistic occurrence of such events. In a voxel-based view, would quantum tunneling still occur, and if so, how?

The concept of quantum superposition, another pillar of quantum mechanics, wherein a quantum system can exist in multiple states simultaneously until measured, could also be impacted. If particles are 3D voxels, can they still exist in a superposition of states, or would this concept need to be radically reinterpreted?

From the perspective of quantum field theory, if particles are 3D voxels, it raises questions about how fields are defined and interact. Currently, particles are viewed as excitations or "quanta" of their respective fields. For instance, a photon is an excitation of the electromagnetic field. What would be the nature of these fields if the quanta were voxel-based? Would this change the nature of fundamental forces?

There would also be consequences for our understanding of symmetries, which play a crucial role in physics. Gauge symmetry underlies the formulation of quantum field theories, and Lorentz symmetry is fundamental to relativity. Would these symmetries hold in a universe composed of 3D voxels?

Similarly, a shift to 3D voxel particles would also influence our understanding of spacetime. Theories of quantum gravity like string theory and loop quantum gravity, which attempt to reconcile quantum mechanics and general relativity, consider spacetime as being smooth or having a granular structure respectively. Would a voxel-based approach advocate a new structure to spacetime itself, and what would be the implications of such a structure?

Answering these questions rigorously demands the development of a comprehensive theoretical framework for voxel-based physics that integrates and extends principles from quantum mechanics, quantum field theory, and general relativity. This, as of now, does not exist.

Thus, the leap from viewing elementary particles as 0-dimensional points or 1-dimensional strings or 2-dimensional membranes to 3-dimensional voxels is not a straightforward one. It demands a radical and as-yet-undeveloped restructuring of fundamental physical theories, which, while exciting to ponder, remains at present purely in the realm of speculation.

## Ⅱ. Connecting Point between YuM-Theory and Existing Physics

We state in this chapter the connection points between YuM-Theory and existing physics, especially quantum mechanics, quantum field theory, and string theory. For the formal foundations, please refer to the Appendix A.

### Quantum Mechanics

Let's attempt to think abstractly about how quantum mechanics might be altered if particles were considered 3D voxels. As a simple example, we might start with the fundamental equation of quantum mechanics, the Schrödinger equation. In its time-dependent form, the equation is:

$$i\hbar\partial\psi/\partial t = -\hbar^2/2m\nabla^2\psi + V\psi$$



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where:
- i is the imaginary unit,
- $\hbar$ is the reduced Planck's constant,
- t is time,
- $\psi$ is the wave function of the system,
- m is the mass of the particle,
- $\nabla^2$ is the Laplacian operator indicating the second spatial derivative,
- and V is the potential energy function.

In this scenario, $\psi$ is a function that provides the probability amplitude for the position of a point-like particle. If we were to consider particles as 3D voxels instead, $\psi$ might need to represent something more complex, like a probability amplitude distribution over a 3D volume rather than at a point in space. This would substantially modify the interpretation and mathematical description of the wave function.

For instance, in the context of voxels, we would define our voxels as the set $v_i$ with i running from $1 \, to \, N$, where N is the total number of voxels. The wavefunction $\psi$ would then become a function defined over the set of voxels, i.e., $\psi(v_i, t)$ instead of $\psi(r, t)$. The Laplacian operator $\nabla^2$, which corresponds to the second spatial derivative, would have to be replaced with a discrete equivalent, taking into account that each voxel $v_i$ has a finite volume.

This is where the complexity arises. Defining the equivalent of a Laplacian for discrete voxel data is non-trivial, and there are several ways this might be done, each with their own mathematical subtleties and assumptions. These definitions would crucially depend on how we model the adjacency and connectivity between voxels.

Moreover, the integral form of the Schrödinger equation, which gives the expected value of a quantum mechanical operator, is given by:

$$\langle O \rangle = \int \psi^* O \psi d^3r$$

Here, $\psi^*$ is the complex conjugate of the wavefunction $\psi$, O is the operator, and $d^3r$ is the volume element. In a voxel-based framework, this would become:

$$\langle O \rangle = \sum_i \psi^*(v_i, t) O \psi(v_i, t) V_i$$

where $V_i$ is the volume of the voxel $v_i$. Again, the definition of the operator O in the context of voxels is non-trivial and would require rigorous formulation.

Likewise, the mathematical framework of quantum field theory, where fields are operator-valued distributions acting on a Fock space, would need substantial revisions to accommodate voxel particles. Field quantization would no longer involve the creation and annihilation of point-like or string-like entities but rather these 3D volumes. □

### Quantum Field Theory

Quantum Field Theory (QFT) represents particles as excitations in fields. The creation and annihilation operators in the field would have to be redefined to act on 3D volumes, rather than points, which would alter the mathematical structure of the quantum fields themselves.

In a voxel-based universe, quantum fields $\varphi(x)$ would now become something like $\varphi(v)$, where v represents a volume element or voxel. The Lagrangian density $\mathscr{L}$, a key concept in QFT, would also need to be redefined in terms of these voxel-based fields.

The canonical commutation relation in QFT, which typically reads:

$$[\varphi(x), \pi(y)] = i\hbar\delta^3(x-y),$$

where $\varphi(x)$ and $\pi(y)$ are the field and its conjugate momentum at points x and y respectively, and $\delta^3(x-y)$ is the 3-dimensional Dirac delta function, might have to be re-expressed in terms of these voxel variables. □

### String Theory

In string theory, particles are seen as one-dimensional strings. The Polyakov action, a fundamental quantity in string theory, is given by:

$$S = -T/2\int d^2\sigma(-h)h^{\alpha}\beta\partial_{\alpha}X^{\mu}\partial_{\beta}X^{\nu}g_{\mu}\nu$$

Here, T is the string tension, $\sigma$ are the parameters on the worldsheet, h is the induced metric on the worldsheet, $X^{\mu}$ are the coordinates of the string in spacetime, and $g_{\mu}\nu$ is the metric tensor of spacetime. In a voxel-based interpretation, the fundamental objects would need to be three-dimensional, requiring a significant change to the mathematical structure of string theory.

Let's take a step back and examine the standard model of particle physics, which treats particles as 0-dimensional entities. The theory relies on mathematical structures like group theory, differential geometry, and functional analysis. For example, the core of the theory can be summarized by the elegant and succinct equation of the Lagrangian:

$$L = -1/4F^{\mu\nu}F_{\mu\nu} + i\bar{\psi}D/\psi - m\bar{\psi}\psi$$

This equation encapsulates the dynamics of the quantum fields that represent particles. The first term relates to the field strength tensor, which encapsulates the electromagnetic, weak, and strong forces. The second term involves the Dirac operator acting on the fermionic field $\psi$, which represents matter particles, and the final term gives the mass of the particles.

If we move from this point-particle picture to a 1-dimensional string picture, the mathematics changes significantly. Instead of quantum field theory, we enter the realm of string theory, which involves advanced mathematics including algebraic geometry, complex analysis, and topology. An example of a fundamental equation in string theory is the Nambu-Goto action, which describes the dynamics of a string:





$$S = -T\int d^2\sigma(-\det(g_{\alpha\beta}))$$

Here, T is the string tension, $\sigma$ represents coordinates on the string's worldsheet, and $g_{\alpha\beta}$ is the induced metric on the worldsheet.

A voxel-based model, treating particles as 3-dimensional entities, would demand even more profound alterations in the mathematical formalism. Each voxel might be represented as a multi-dimensional matrix rather than a point or a 1-dimensional string. This could potentially require the use of non-commutative geometry, topological field theories, or other mathematical frameworks not typically employed in current particle physics. □

## III. YuM-Theory Category Theoretic Formulation

We state in this chapter YuM-Theory and from a category theory perspective. For the formal foundations, please refer to the Appendix B.

Category theory is a branch of abstract mathematics with broad applications in various areas of study including physics, computer science, and of course, mathematics itself. The core concepts in category theory are "objects" and "morphisms". Objects are usually mathematical structures, and morphisms are maps between these structures preserving certain properties. For instance, in the category of sets (Set), objects are sets and morphisms are functions.[9]

In theoretical physics, category theory is often used in algebraic topology, quantum mechanics, and quantum field theory. But its utilization in trying to restructure the standard model or string theory into a voxel-based approach, treating particles as 3-dimensional entities, is largely unexplored territory.

Nonetheless, we can think of the theory in the following way: we can consider a category of "voxels", where the objects in this category are 3D cubes (voxels) in our hypothetical voxel-space. Morphisms in this category could be considered as transformations between these voxels, preserving some as-yet-undefined quantum mechanical properties.

Key concepts in category theory, such as Kan extensions and limits/colimits, could be used to capture the idea of "nearness" or "continuity" in voxel-space, as well as to define a sense of transformations between different voxel configurations:

In order to provide mathematical formulas that correspond to these categorical concepts, let's denote by C and D our categories of interest, and let's denote functors between these categories by F and G. In our case, C could be a category whose objects are 'voxels' and whose morphisms are some sort of 'relations' between these voxels.

**1. Kan Extensions:** These provide a way of extending diagrams along a functor. In our context, we might imagine a functor that assigns to each voxel some quantum mechanical system (a Hilbert space, for instance). A Kan extension could then be used to understand how the quantum systems assigned to "nearby" voxels are related to each other. Given a functor $F : C \to D$, a right Kan

extension of F along $G : C \to E$ is a functor $\mathrm{Ran}_G F : E \to D$ together with a natural transformation $\varepsilon : \mathrm{Ran}_G FG \Rightarrow F$ such that for every functor $H : E \to D$, there exists a unique natural transformation $\eta : H \Rightarrow \mathrm{Ran}_G F$ making the following diagram commute:

$H \Rightarrow \mathrm{Ran}_G F$
$\downarrow \quad \downarrow$
$F \Rightarrow \mathrm{Ran}_G FG$

Here the downwards arrows are ε's under H and G respectively.

**2. Limit and Colimit:** Limits (or colimits) capture the notion of the "smallest" (or "largest") object in a category that stands in a specific relationship to a set of other objects. For instance, a limit could represent the "intersection" of two voxels, if such a notion is meaningful in our theory. Let's consider a diagram $F : J \to C$ from a small category J to our category of interest C. A cone to F is an object N of C together with a family of morphisms $\varphi : N \to F(j)$ in C such that for every morphism $f : j \to j'$ in J, we have $F(f) \circ \varphi = \varphi'$. The limit of the diagram F is a cone $(L, \psi)$ to F such that for every other cone $(N, \varphi)$ to F there exists a unique morphism $u : N \to L$ in C such that $\psi \circ u = \varphi$. Colimits are defined dually.

**3. Preserving Extensions:** A functor between categories is said to preserve limits (or colimits) if it sends limits in the source category to limits in the target category. In our voxel theory, such functors might describe transformations of the whole voxel-space, and preserving extensions would ensure that these transformations respect the "nearness" or "intersection" relationships between voxels. A functor $F : C \to D$ is said to preserve limits (or colimits) if for every diagram $G : J \to C$, the limit of G (if it exists in C) is sent by F to the limit of $F \circ G$ in D. Similarly, F preserves colimits if every colimit in C is sent to a colimit in D.

**4. Pointwise Kan Extensions:** Pointwise Kan extensions are a particular way of computing Kan extensions that use limits or colimits. If our functor assigns to each voxel a quantum mechanical system, a pointwise Kan extension could describe how to create a "new" quantum system out of an "old" one, in a way that respects the nearness relationships between voxels. The right Kan extension $\mathrm{Ran}_{GF}$, when it exists, can be computed pointwise. For each object e in E, we take the limit over the comma category $(G \downarrow e)$ of the composite of F with the projection functor $p : (G \downarrow e) \to C$. This gives $(\mathrm{Ran}_{GF})(e)$ as the limit.

**5. Density:** In category theory, the density theorem provides a way to represent a category as a category of sheaves on a different category. In our context, this might provide a means to represent our voxel-space as a more familiar space (like a manifold), with additional structure given by the "density" of voxels. Given a full subcategory J of C, every object X of C is the colimit of the diagram $\mathrm{Hom}_C(-, j)$ for j running over the objects of J. This is the statement of the density theorem.





**6. Formal Category Theory:** This involves applying category theory to itself. In the voxel model, this could potentially involve creating categories whose objects are voxel-spaces themselves, with morphisms given by certain types of transformations of voxel-spaces. Formal category theory deals with notions such as functor categories $[C, D]$ of all functors from C to D. It also includes the study

of 2-categories, where there are 'morphisms between morphisms'. The idea of transformations between voxel-spaces might lead to a 2-category, with voxel-spaces as objects, transformations as 1-morphisms, and 'relations between transformations' as 2-morphisms.

The formulas for these constructs can be quite involved and rely heavily on categorical language. The challenging task will be to interpret these mathematical constructs in a way that leads to a meaningful physical theory of elementary particles and voxels.

# IV. Conclusion and Future Work

In conclusion, the voxel-based approach to understanding elementary particles presents a novel perspective on the architecture of the universe. We have used tools from string theory, higher-dimensional spacetime considerations, and category theory to frame our approach and provide an initial outline of the theoretical constructs that may be involved. This approach expands the concept of particles from 0-dimensional points to 3-dimensional voxels, and suggests that these voxels could potentially serve as the fundamental building blocks of the universe.

Several key ideas have emerged from our discussions. We've described elementary particles as 3-dimensional voxels and the universe as a voxel-space. We've employed brane-theory from string theory and used categorical methods to provide mathematical structures for these concepts. We've also offered a preliminary discussion of how these categorical methods could be relevant in the context of a voxel-based theory of particles.

However, our exploration thus far is only the first step in a much larger endeavor. Numerous questions remain to be addressed and challenges to be resolved in order to fully realize the potential of the voxel-based approach.

For future work, we will need to address several key areas:

**1. Detailed Physical Interpretation:** How do the categorical structures we have discussed correspond to physical processes and principles in the real world? How do we interpret concepts like Kan extensions, limits, and colimits in physical terms?

**2. Quantum Mechanics and Field Theory:** How can voxel-based approaches be integrated with quantum mechanics and quantum field theory? How does this affect our understanding of fundamental forces and interactions between particles?

**3. Empirical Testability:** How can the voxel-based approach be made testable? What predictions does it make that can be confirmed or refuted by experiments?

**4. Unification with Existing Theories:** How does the voxel-based approach fit in with existing physical theories like quantum mechanics, quantum field theory, general relativity, and string theory?

**5. Mathematical Rigor:** As we have used a lot of high-level mathematical concepts, more work needs to be done to make the mathematics rigorous, and to ensure that the theory is mathematically consistent.

The voxel-based approach to elementary particles is a new frontier in theoretical physics, and much work remains to be done to fully explore this concept. The exciting promise of this approach lies in its potential to provide a more unified and comprehensive understanding of the universe at its most fundamental level. The journey to that understanding, however, will require substantial effort, creativity, and a willingness to explore new and uncharted territories in both physics and mathematics.

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# VI. Appendix A

**Definition 1 (Voxel)**: A voxel v is a discrete unit of three-dimensional volume in a larger space, often represented mathematically as an element in a three-dimensional grid G (i.e., $G = v_i | i \in I$ where I is an index set).

**Definition 2 (Voxel Space):** A voxel space V is a set of all voxels that constitute a larger volume, typically represented as a three-dimensional array or a mathematical lattice.

**Theorem 1 (Voxel Hypothesis):** Each elementary particle is a 3-dimensional voxel within the space-time fabric of the universe.

**Lemma 1 (Discretization):** The voxel representation of elementary particles implies a discrete structure of space-time.

*Proof.* If each elementary particle is associated with a voxel, and if we assume that these voxels are indivisible and occupy a fixed volume of space, then the implication is a digital or discrete space-time, where the smallest unit is one voxel.

**Theorem 2 (Voxel Dynamics):** The evolution of elementary particles in voxel space can be described by a certain set of computational rules or dynamics.

**Theorem 3 (Voxel-State Correspondence):** There is a correspondence between the state of an elementary particle and the information content of its associated voxel.

**Definition 3 (Quantum State on a Voxel):** For each voxel v in a voxel space V, a quantum state $|v>$ is associated, forming a Hilbert space H. The quantum state of a voxel v, $|v>$, belongs to the Hilbert space (i.e., $|v> \in$ H).

**Definition 4 (Time Evolution of Quantum State in Voxel Space):** The time evolution of a quantum state $|\psi> \in$ H in voxel space is described by a discrete version of the Schrödinger equation:

$$|\psi(t+\Delta t)> = U|\psi(t)>$$

Here, U is a unitary operator that acts on the state $|\psi>$ in the Hilbert space H, $\Delta$t is the discrete time step, and $|\psi(t+\Delta t)>$ and $|\psi(t)>$ are the states of the system at times t $+ \Delta$t and t, respectively.

**Definition 5 (Voxelized Schrödinger Equation):** The voxelized Schrödinger Equation is a differential equation of the form iħ ∂/∂t |ψ(v, t)> = Ô |ψ(v, t)>, where |ψ(v, t)> is the quantum state of a voxel v at time t, and Ô is the voxelized quantum field operator acting on $|\psi(v,t)>$.

**Definition 6 (Voxelized Quantum Fluctuation):** A voxelized quantum fluctuation at a voxel $v \in V$ is represented as a change $\Delta|\psi>$ in the quantum state $|\psi(v)>$ of v. This can be modeled as the action of a fluctuation operator $\hat{O}_F$ on $|\psi(v)>$, i.e., $\Delta|\psi> = \hat{O}_F|\psi(v)>$.

**Definition 7 (Voxelized Quantum Vacuum):** A voxelized quantum vacuum is a special quantum state $|0>$ such that for any voxel v $\in$ V, $\Phi(v) = |0>$, i.e., the quantum field assigns the vacuum state to every voxel.

**Theorem 4 (Voxelized Schrödinger Evolution):** Let $|\psi(v,t)>$ be a state of voxel v at time t evolving under the voxelized Schrödinger equation $i\hbar\partial/\partial t|\psi(v,t)> = \hat{O}|\psi(v,t)>$. Then, for any given initial state $|\psi(v,0)>$ at $t=0$, there exists a unique state $|\psi(v,t)>$ for all $t>0$.

*Proof:* We can regard the voxelized Schrödinger equation as a first-order linear differential equation for the function $|\psi(v,t)>$. Uniqueness and existence of the solution for such equations are standard results in the theory of differential equations.

**Lemma 2 (Voxelized Quantum Vacuum Stability):** The voxelized quantum vacuum state $|0>$ is a stationary state of the voxelized Schrödinger equation.

*Proof:* By definition, the voxelized quantum field operator Ô acting on the vacuum state $|0>$ gives 0. Therefore, the voxelized Schrödinger equation $i\hbar\partial/\partial t|0> = \hat{O}|0>$ reduces to $\partial/\partial t|0> = 0$, showing that $|0>$ does not change with time.



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**Theorem 5 (Voxelized Quantum Fluctuation Persistence):** Let $|\psi(v)>$ be a state of a voxel v and let $\Delta|\psi> = \hat{O}_F|\psi(v)>$ represent a quantum fluctuation. Then, the effect of the fluctuation persists in the time evolution of the state, i.e., if $|\psi'(v,0)> = |\psi(v)> + \Delta|\psi>$, then $|\psi'(v,t)> \neq |\psi(v,t)>$ for all $t>0$.

*Proof:* The proof of this theorem would be a direct application of the linearity of the Schrödinger equation and the non-zero action of the fluctuation operator $\hat{O}_F$ on the quantum state $|\psi(v)>$.

**Definition 8 (Voxelized Quantum Field):** A voxelized quantum field $\Phi$ is a map from the voxel space V to a Hilbert space H, denoted as $\Phi$ : V → H. For each voxel v ∈ V, $\Phi(v)$ corresponds to a quantum state in the Hilbert space H.

**Definition 9 (Voxelized Quantum Field Operator):** Given a voxelized quantum field $\Phi$ : V → H, a voxelized quantum field operator Ô is a linear operator that acts on the quantum field $\Phi$, transforming it to another quantum field. If we denote the action of Ô on $\Phi$ by $\hat{O}\Phi$, then for each voxel v ∈ V, $(\hat{O}\Phi)(v) = \hat{O}(\Phi(v))$ is a new quantum state in the Hilbert space H.

**Lemma 3 (Commutation Relations):** The voxelized quantum field operators obey the commutation relations:

$[a_v, a_w†] = \delta_{vw}I$,
$[a_v, a_w] = [a_v†, a_w†] = 0$,

for all voxels v, w in V. Here, $\delta_{vw}$ is the Kronecker delta, and I is the identity operator on F.

*Proof:* This is a standard result in QFT when the field operators are interpreted as creation and annihilation operators. These relations ensure that the quantum field exhibits the correct quantum statistics.

**Theorem 6 (Representation Theorem):** Any state in the Fock space F can be generated from the vacuum state $|0>$ by the action of the creation operators $a_v†$.

*Proof:* This follows directly from the properties of creation and annihilation operators in quantum field theory. The vacuum state is defined as the state that is annihilated by all the annihilation operators: $a_v|0> = 0$ for all voxels v. Then, the action of $a_v†$ on $|0>$ creates a single "particle" in voxel v. By repeated action of the creation operators, any state in F can be produced.

**Definition 10 (Voxelized String State):** A voxelized string state is a state in the Hilbert space H which is an eigenstate of the voxelized quantum field operators $\Phi(V)$ for a certain set of voxels.

**Theorem 7 (Voxel String Duality):** Let $|\Psi\rangle$ be a voxelized string state. Then, for any set of voxels $V_i$, there exists a one-dimensional string state $|\xi\rangle$ in standard string theory such that $\langle\Psi|\Phi(V_i)|\Psi\rangle = \langle\xi|\Phi_s(\xi_i)|\xi\rangle$ for all i, where $\Phi_s$ is the string theory quantum field operator, and $\xi_i$ is a point on the string corresponding to the voxel $V_i$.

Let's dive into the three-dimensional nature of voxels and its implications on the very fabric of space-time and quantum mechanics:

**Definition 11 (Voxelized Space-Time Lattice):** Given the universe U, we define it as a three-dimensional lattice L such that:

$$U = \bigcup_{i \in \mathbb{N}} v_i$$

Where each $v_i$ is a voxel with a finite three-dimensional volume $\Delta V$.

**Lemma 4 (Finite Granularity of Space):** Space itself has a minimal, indivisible volume unit, $\Delta V$, which is the volume of a voxel.

*Proof:* From Definition 1, a voxel is the smallest, indivisible unit of space. Thus, $\Delta V$ represents the smallest granularity of space possible.

**Theorem 8 (Space-Time Quantization):** Space-time is inherently quantized, with each voxel representing a quantum unit of space.

*Proof:* This follows directly from Lemma 6. Given that space has a minimum volume granularity, it implies that all physical quantities and fields are inherently discrete at scales of $\Delta V$.

**Definition 12 (Voxel Field States):** Within each voxel $v_i$, quantum fields are represented not as continuous functions but as sets of quantized states. A quantum field $\Phi$ in voxel $v_i$ is defined as:

$$\Phi_{v_i} = \{\phi_{ij} \mid j \in \mathbb{N}\}$$

Where each $\phi_{ij}$ is a quantized field state within the voxel.

**Lemma 5 (Boundary Interactions):** Given two adjacent voxels $v_i$ and $v_j$, the quantum states at the boundary will interact. This interaction forms the basis for particle interactions and dynamics in the voxel space.

*Proof:* Due to the finite volume of voxels and the quantized nature of fields within them, field states at the boundary of two adjacent voxels will inevitably interact, leading to observable particle interactions.

**Theorem 9 (Holographic Principle in Voxel Space):** The total information or entropy contained within a given volume can be represented on the surface area of that volume, with each voxel on the surface representing a quantum bit or "qubit" of information.

*Proof:* Given that each voxel represents a quantum unit of space and contains quantized field states, the maximum entropy or information a voxel can contain is finite. As a result, for a given volume, the maximum entropy is proportional to its surface area, leading to a voxel-based version of the holographic principle.





**Corollary 4 (Limit to Information Density):** There exists a maximum limit to the information density in space-time, determined by the volume $\Delta V$ of a voxel.

**Remark:** The three-dimensional nature of voxels fundamentally alters the continuum understanding of quantum mechanics and general relativity. Space is not smooth but granular, leading to novel implications on quantum gravity, entropy, and the very nature of reality.

**Definition 13 (Voxel Architecture):** A voxel, by nature, can be represented as a three-dimensional matrix. Given a voxel v in our universe, it can be represented as:

$$v = M_{3\times3\times3}(a_{ijk})$$

Where M is the matrix representation of the voxel, and $a_{ijk}$ are elemental data points contained within.

**Theorem 10 (Three-Dimensional Interactions):** Each elemental data point $a_{ijk}$ within a voxel interacts not just with its immediate neighbors in the traditional 3D space, but in all three dimensions, meaning its interactions span more complex patterns than previously thought in elementary particles.

*Proof:* Given the voxel architecture from Definition 13, each elemental point $a_{ijk}$ has potential interactions in the x, y, and z dimensions. This is intrinsic to the three-dimensional nature of a voxel.

**Lemma 6 (Voxel Stability):** A voxel's stability, and hence the stability of an elementary particle, depends on the stability of the majority of its elemental data points in the matrix.

**Definition 14 (Voxel-Force Tensor):** Given the three-dimensional interactions of elemental data points within a voxel, the forces at play within a voxel can be represented as a tensor, T, where each tensor component represents forces in the respective dimensions.

$$T_{ijk} = f(a_{ijk})$$

Where f is the force function for a given elemental data point.

**Theorem 11 (Unified Voxel Dynamics):** The dynamics of an elementary particle, modeled as a voxel, can be holistically described by the dynamics of its elemental data points and their interactions in the three-dimensional matrix.

*Proof:* From Theorem 10 and the Voxel-Force Tensor definition, it's clear that the entirety of a voxel's dynamics are determined by the forces and interactions of its elemental data points.

**Proposition (Voxel Wave Function):** Each voxel can be associated with a wave function, which is a result of the superposition of the wave functions of all its elemental data points.

$$\Psi(v) = \sum_{i,j,k} \psi(a_{ijk})$$

Where $\psi$ is the wave function for an elemental data point.

**Remark:** The granularity introduced by voxel-based elementary particles redefines space not as a continuum but as a mosaic of interacting 3D matrices. This granularity provides novel insights into the fabric of space-time, allowing for a reinterpretation of phenomena like entanglement, superposition, and even quantum gravity.

**Corollary 5 (Quantum Voxel Entanglement):** Given two elementary particles represented as voxels, if any of their respective elemental data points are entangled, the voxels themselves can be considered entangled.

**Lemma 7 (Dimensional Reduction):** The overall properties of a voxel can be derived from observing the patterns and interactions of its elemental data points in a reduced one-dimensional representation.

This voxel-based interpretation paints a picture of the universe as a vast collection of interacting 3D matrices. The inherent three-dimensionality of these voxels provides a structured, granular framework that could offer novel solutions and interpretations to longstanding problems in quantum mechanics and relativity.

# VII. Appendix B

**Definition 1 (Category):** A category C consists of a class $\mathrm{Ob}(C)$ of objects, a class $\mathrm{Hom}(C)$ of morphisms, or arrows, between the objects. For every two objects A, B in $\mathrm{Ob}(C)$, we have a set $\mathrm{Hom}(A, B)$ in $\mathrm{Hom}(C)$, each of whose elements is a morphism $f : A \to B$. This comes with two operations, namely “composition” and “identity”.

*Composition:* $\forall A, B, C \in \mathrm{Ob}(C)$, there exists a binary operation $\circ : \mathrm{Hom}(B, C) x \mathrm{Hom}(A, B) \to \mathrm{Hom}(A, C)$, called the composition operation.

*Identity:* $\forall A \in \mathrm{Ob}(C)$, there exists a morphism $id_A : A \to A$, called the identity morphism on A.

These operations satisfy the following laws:

- Associativity: $\forall f \in \mathrm{Hom}(A, B)$, $\forall g \in \mathrm{Hom}(B, C)$, $\forall h \in \mathrm{Hom}(C, D)$, $(h \circ g) \circ f = h \circ (g \circ f)$.
- Identity: $\forall f \in \mathrm{Hom}(A, B)$, $f \circ id_A = f$ and $id_B \circ f = f$.

**Definition 2 (Functor):** A Functor F from category C to category D, denoted $F : C \to D$, is a mapping that associates each object A in C with an object $F(A)$ in D and each morphism $f : A \to B$ in C with a morphism $F(f) : F(A) \to F(B)$ in D such that the following two conditions are satisfied:





1. $F(g \circ f) = F(g) \circ F(f)$ for all morphisms $f : A \to B$ and $g : B \to C$ in C.
2. $F(id_A) = id_F(A)$ for every object A in C.

**Definition 3 (Natural Transformation):** Given two functors $F, G : C \to D$, a natural transformation $\eta : F \to G$ is a family of morphisms in D such that for each object X in C, we have a morphism $\eta_X : F(X) \to G(X)$ and for every morphism $f : X \to Y$ in C, the following “naturality square” commutes:

$$\eta_Y \circ F(f) = G(f) \circ \eta_X$$

**Definition 4 (Kan Extensions):** Given a functor $F : C \to D$ and a functor $G : C \to E$, a right Kan extension of F along G is a functor $\mathrm{Ran}_G F : E \to D$ and a natural transformation $\varepsilon : \mathrm{Ran}_G F \circ G \Rightarrow F$. For every e in E, we define:

$$(\mathrm{Ran}_G F)(e) = \lim((G \downarrow e) \to C, F)$$

where $(G \downarrow e) \to C$ is the comma category, the limit is taken over this category, and F is then applied. $\varepsilon$ is the natural transformation whose components are the universal co-cones.

**Definition 5 (Limit and Colimit):** The limit of a diagram $F : J \to C$, for J a small category, is an object lim F in C and a natural transformation $u : \Delta(\lim F) \Rightarrow F$ where $\Delta : C \to C^J$ is a diagonal functor. The colimit is defined dually.

**Definition 6 (Preserving Extensions):** A functor $F : C \to D$ is said to preserve limits if for every diagram $G : J \to C$, if the limit of G exists in C, then F sends this limit to the limit of $F \circ G$ in D. Colimits are defined dually.

**Definition 7 (Density):** Given a full subcategory J of C, the density theorem states that every object X in C is the colimit of the diagram $\mathrm{Hom}_C(-, j)$ for j running over the objects of J.

**Definition 8 (Formal Category Theory):** Functor category $[C, D]$ is a category whose objects are all functors from C to D and morphisms are natural transformations between these functors. In a 2-category, there are ‘morphisms between morphisms’, i.e., for two 1-morphisms $f, g : A \to B$, there is a set $\mathrm{hom}(f, g)$ of 2-morphisms.

**Definition 9 (Voxel Category):** A voxel category V is a category in which the objects are 3-dimensional cube-like entities (voxels) and the morphisms are transformations between these voxels preserving certain quantum mechanical properties. Formally, for any two objects $A, B \in \mathrm{Ob}(V)$, the set of morphisms $\mathrm{Mor}(A, B)$ in V is a set of transformations between these voxels preserving quantum mechanical properties.

**Theorem 1 (Voxel Functor Preservation):** Suppose $F : V \to W$ is a functor from voxel category V to another category W. If F preserves limits and colimits, then for every diagram $G : J \to V$, if the limit (or colimit) of G exists in V, then F sends this limit (or colimit) to the limit (or colimit) of $F \circ G$ in W. This property ensures that the transformations respect the “nearness” or “intersection” relationships between voxels.

**Lemma 1 (Voxel Kan Extensions):** Given a functor $F : V \to W$ and a functor $G : V \to E$, a right Kan extension of F along G provides a means to understand how the quantum systems assigned to "nearby" voxels are related to each other. The Kan extension exists if E has all the required limits.

**Definition 10 (Voxel Density):** For a full subcategory J of V, the density of an object X in V is defined as the colimit of the diagram $\mathrm{Hom}_V(-, j)$ for j running over the objects of J. This might provide a means to represent our voxel-space as a more familiar space (like a manifold), with additional structure given by the "density" of voxels.

**Theorem 2 (Voxel Formal Category Theory):** A voxel functor category $[V, W]$ is a category whose objects are all functors from V to W and morphisms are natural transformations between these functors. In a 2-category of voxel spaces, there are 'morphisms between morphisms', which could potentially involve creating categories whose objects are voxel-spaces themselves, with morphisms given by certain types of transformations of voxel-spaces.

**Definition 11 (Pointwise Kan Extensions):** Let V, W, and E be categories, and $F : V \to W$, $G: V \to E$ be functors. If W has all limits indexed by a comma category $(G \downarrow e)$ for each $e \in \mathrm{Ob}(E)$, then the right Kan extension $\mathrm{Ran}_G F$ of F along G can be computed pointwise as follows:

$$(\mathrm{Ran}_G F)(e) = \mathrm{Lim}_{(G \downarrow e)} F \circ p$$

where $p : (G \downarrow e) \to V$ is the projection functor.

**Lemma 2 (Existence of Voxel Kan Extensions):** Given functors $F : V \to W$ and $G : V \to E$ from a voxel category V to categories W and E respectively, a right Kan extension of F along G exists if E has all the limits indexed by comma categories $(G \downarrow e)$ for each $e \in \mathrm{Ob}(E)$.

Proof of this lemma is based on the definition of pointwise Kan extensions and the universal property of limits in E.

**Theorem 3 (Density Theorem in Voxel Categories):** Given a full subcategory J of a voxel category V, every object X of V is the colimit of the diagram $\mathrm{Hom}_V(-, j)$ for j running over the objects of J. This is the statement of the density theorem in voxel categories. Formally,

$$X = \mathrm{Colim}_j \mathrm{Hom}_V(-, j)$$

**Definition 12 (Voxel Sheaf):** A sheaf $(\mathscr{F})$ on a voxel category $(V)$ assigns to each voxel $(v \in V)$ a quantum mechanical system, such that for any collection of overlapping voxels, their associated quantum systems are locally consistent. Formally, this means that if $(v_i)$ is a collection of voxels in $(V)$ with intersections $(v_i \cap v_j)$, then the system associated to $(v_i \cap v_j)$ by $(\mathscr{F})$ is the restriction of both the systems associated to $(v_i)$ and $(v_j)$ by $(\mathscr{F})$.

**Lemma 3 (Voxel Sheaf Morphisms):** Let $(\mathscr{F})$ and $(\mathscr{G})$ be two voxel sheaves on a voxel category $(V)$. A morphism between these sheaves is a natural transformation $(\eta : \mathscr{F} \Rightarrow \mathscr{G})$ that preserves the local consistency property on overlapping voxels.





**Theorem 4 (Equivalence of Voxel Categories):** Let $(V)$ and $(W)$ be two voxel categories. If there exists a pair of functors $(F : V \to W)$ and $(G : W \to V)$ such that $(F \circ G \cong \mathrm{Id}_W)$ and $(G \circ F \cong \mathrm{Id}_V)$, then $(V)$ and $(W)$ are equivalent as voxel categories.

*Proof:* This result is a direct consequence of the definition of equivalence of categories, with the understanding that $(V)$ and $(W)$ are voxel categories, hence the morphisms preserve the specified voxel properties.

**Definition 13 (Grothendieck Topology on Voxel Category):** A Grothendieck topology $(J)$ on a voxel category $(V)$ is a collection of families of morphisms (called covering sieves) satisfying certain axioms. These sieves define how voxels "cover" other voxels, thus establishing a notion of "locality" in the voxel category.

**Lemma 4 (Existence of Grothendieck Topologies):** Every voxel category admits a Grothendieck topology, possibly trivial, that respects the categorical structures inherent to the voxel category.

**Theorem 5 (Sheafification):** For any presheaf $(\mathscr{P})$ on a voxel category $(V)$ with a Grothendieck topology $(J)$, there exists a unique (up to isomorphism) sheaf $(\mathscr{F})$ and a morphism $(m : \mathscr{P} \to \mathscr{F})$ such that for any other morphism $(n : \mathscr{P} \to \mathscr{G})$ to a sheaf $(\mathscr{G})$, there exists a unique morphism $(u : \mathscr{F} \to \mathscr{G})$ such that $(u \circ m = n)$.