Boundary Spectral Process Categories: Observable Dynamics, Gauge Redundancy, and Topological Quantum Codes

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We introduce boundary spectral process categories, a finite-dimensional categorical framework for describing quantum dynamics through the functorial transport of boundary-supported spectral data. The framework is designed for systems in which the physically accessible observables are localized on, or reconstructed from, a boundary interface rather than assigned to a naive tensor factor of a bulk Hilbert space. The central mathematical structure is a dagger symmetric monoidal category equipped with boundary systems, spectral projector functors, and causally admissible morphisms that transport observable decompositions at the projector level. This additional projector-level structure removes an ambiguity present in formulations that transport only abstract observables. We prove a categorical boundary Born rule, a finite-dimensional representation-dependent Stone theorem for reversible boundary flows, a non-circular gauge-invisibility theorem, and a boundary formulation of quantum error correction. A toric-code example shows how local bulk errors act trivially on logical boundary spectral data below the code distance, while noncontractible errors induce nontrivial boundary symmetries.

INTRODUCTION

Categorical quantum mechanics reformulates quantum theory in terms of objects, morphisms, tensor products, dagger structure, and graphical composition. Abramsky and Coecke showed that a large part of finite-dimensional quantum protocol theory, including teleportation and a categorical form of the Born rule, can be expressed in dagger compact categories with biproducts [1]. Selinger's CPM construction provided a categorical account of completely positive maps [2]. Later developments of process theories and diagrammatic calculi clarified causality, classical structure, measurement, and quantum channels in increasingly general categorical settings [3–6].

Many physical systems of current interest are not naturally described by assigning all observable content to an unconstrained bulk tensor factor. Gauge theories, topological phases, holographic toy models, and quantum error-correcting codes often distinguish between microscopic degrees of freedom and physically accessible or protected observables. In such systems, the operative question is not simply what the Hilbert space is, but which observables are available at the boundary interface and how their spectral decompositions are transported by admissible processes.

This paper develops a categorical formalism for that question. The guiding idea is that boundary observables should not be represented merely by labels or abstract vector-space elements. They should carry their spectral projector data, and admissible physical processes should transport that data coherently. This leads to the notion of a boundary spectral process category. The additional projector-level transport is essential. Without it, a statement such as "a process acts trivially on boundary observables" does not automatically imply that it preserves outcome probabilities. Here that implication becomes a theorem rather than an assumption.

The main contributions are as follows: (i) a definition of boundary spectral process categories with explicit boundary systems and projector-level spectral transport; (ii) a categorical Born rule for boundary spectral measurements; (iii) a finite-dimensional representation-dependent Stone theorem for reversible boundary flows; (iv) a non-circular theorem identifying gauge redundancy with operational invisibility; and (v) a categorical formulation of protected logical observables in stabilizer-type topological codes, illustrated by the toric code.

The results are finite-dimensional throughout. This is intentional. The purpose is to isolate the algebraic and categorical content without introducing unbounded operators, domain questions, or infinite-dimensional spectral subtleties.

PRELIMINARIES

Let $\mathcal{C}$ be a dagger symmetric monoidal category with monoidal unit I, tensor product $\otimes$, and dagger $(-)^\dagger$. A state of an object A is a morphism $\psi : I \to A$, an effect is a morphism $e : A \to I$, and a scalar is an endomorphism $s : I \to I$. We assume finite dagger biproducts whenever finite sums of morphisms are used.

A projector on A is an endomorphism $P : A \to A$ satisfying

$$P^\dagger = P, \qquad P^2 = P. \tag{1}$$

Two projectors $P, Q : A \to A$ are orthogonal when $PQ = QP = 0$. A finite projective decomposition of the identity on A is a finite family of projectors $\{P_i : A \to A\}_{i\in X}$ such that

$$P_i P_j = 0 \quad (i \neq j), \qquad \sum_{i\in X} P_i = 1_A. \tag{2}$$

The category **FdHilb** of finite-dimensional Hilbert spaces and linear maps is the intended representation category, but the abstract definitions are given before choosing a Hilbert representation.

BOUNDARY SPECTRAL DATA

Definition 1 (Boundary object assignment). *A boundary object assignment on $\mathcal{C}$ consists of: for every physical system A,* *a boundary object $\partial A \in \mathcal{C}$; a boundary restriction morphism $\rho_A : A \to \partial A$; and, for every pair A, B, a coherent comparison morphism $\partial(A \otimes B) \to \partial A \otimes \partial B$.*

The object ∂A is interpreted as the system carrying boundary-accessible information. The morphism ρ_A is not required to be unitary or invertible. It may represent restriction, coarse-graining, or logical decoding.

Definition 2 (Boundary spectral observable)**.** *A boundary spectral observable on A is a pair*

$$O = (X_O, \{P_x^O : \partial A \to \partial A\}_{x \in X_O}), \tag{3}$$

where X_O is a finite outcome set and $\{P_x^O\}_{x \in X_O}$ is a finite projective decomposition of the identity on ∂A. Let $\mathsf{BSpec}(A)$ denote the collection of boundary spectral observables on A.

Given $O = (X_O, \{P_x^O\})$ on A, define its bulk effect projectors by

$$\widetilde{P}_x^O := \rho_A^\dagger P_x^O \rho_A : A \to A. \tag{4}$$

These need not form a projective decomposition of 1_A unless ρ_A is an isometry. This distinction is physically important: boundary measurements may be coarse-grained relative to the microscopic bulk.

BOUNDARY SPECTRAL PROCESS CATEGORIES

Definition 3 (Boundary spectral process category)**.** *A boundary spectral process category consists of data*

$$\mathbb{B} = (\mathcal{C}, \otimes, I, (-)^\dagger, \partial, \rho, \mathsf{Cau}, \mathsf{BSpec}, \mathsf{T}) \tag{5}$$

such that:

1. *$\mathcal{C}$ is a dagger symmetric monoidal category with finite dagger biproducts.*
2. *∂ is a boundary object assignment.*
3. *$\mathsf{Cau} \subseteq \mathcal{C}$ is a wide dagger subcategory of causally admissible morphisms.*
4. *For each object A, $\mathsf{BSpec}(A)$ is the set of boundary spectral observables on A.*
5. *For every causal morphism $f : A \to B$, there is a boundary transport map $\mathsf{T}_f : \mathsf{BSpec}(A) \to \mathsf{BSpec}(B)$.*
6. *Transport is functorial: $\mathsf{T}_{1_A} = 1_{\mathsf{BSpec}(A)}$ and $\mathsf{T}_{g \circ f} = \mathsf{T}_g \circ \mathsf{T}_f$.*
7. *For reversible causal $f : A \to B$, there exists a boundary unitary $\partial f : \partial A \to \partial B$ such that, for each spectral projector P_x, $\mathsf{T}_f(P_x) = (\partial f) P_x (\partial f)^\dagger$.*
8. *Boundary restriction is compatible with reversible causal processes: $\rho_B f = (\partial f) \rho_A$.*

The seventh condition is the key structural requirement. It says that reversible physical processes transport spectral projectors by boundary conjugation. This is stronger than merely transporting an abstract observable label and ensures that probability preservation theorems can be proved directly.

BOUNDARY BORN RULE

Definition 4 (Boundary-normalized state)**.** *A state $\psi : I \to A$ is boundary-normalized if*

$$(\rho_A \psi)^\dagger (\rho_A \psi) = 1_I. \tag{6}$$

Equivalently, the induced boundary state $\psi_\partial := \rho_A \psi : I \to \partial A$ is normalized.

Definition 5 (Boundary probability)**.** *Let $O = (X, \{P_x\}_{x \in X})$ be a boundary spectral observable on A. For a boundary-normalized state $\psi : I \to A$, define*

$$p_\psi^O(x) := (\rho_A \psi)^\dagger P_x (\rho_A \psi). \tag{7}$$

Theorem 1 (Boundary Born rule)**.** *Let $\mathbb{B}$ be a boundary spectral process category. Let $O = (X, \{P_x\}_{x \in X})$ be a boundary spectral observable on A, and let $\psi : I \to A$ be boundary-normalized. Then $p_\psi^O(x)$ is positive for every $x \in X$, and*

$$\sum_{x \in X} p_\psi^O(x) = 1_I. \tag{8}$$

Proof. Fix $x \in X$. By definition,

$$p_\psi^O(x) = (\rho_A \psi)^\dagger P_x (\rho_A \psi). \tag{9}$$

Since P_x is a projector, $P_x^\dagger = P_x$ and $P_x^2 = P_x$, hence $P_x = P_x^\dagger P_x$. Therefore

$$p_\psi^O(x) = (\rho_A \psi)^\dagger P_x^\dagger P_x (\rho_A \psi) = (P_x \rho_A \psi)^\dagger (P_x \rho_A \psi), \tag{10}$$

which has the form $r^\dagger r$. Thus it is positive.

For normalization, use $\sum_{x \in X} P_x = 1_{\partial A}$:

$$\sum_{x \in X} p_\psi^O(x) = \sum_{x \in X} (\rho_A \psi)^\dagger P_x (\rho_A \psi) \tag{11}$$

$$= (\rho_A \psi)^\dagger \left(\sum_{x \in X} P_x \right) (\rho_A \psi) \tag{12}$$

$$= (\rho_A \psi)^\dagger (\rho_A \psi) = 1_I, \tag{13}$$

where the last equality uses boundary normalization. This proves the claim. □

REVERSIBLE BOUNDARY SPECTRAL FLOWS

Definition 6 (Reversible boundary spectral flow)**.** *A reversible boundary spectral flow on A is a family of reversible causal morphisms $U_t : A \to A$, $t \in \mathbb{R}$, such that*

$$U_0 = 1_A, \quad U_{t+s} = U_t U_s, \quad U_t^\dagger = U_{-t}. \tag{14}$$

It induces a boundary flow $\partial U_t : \partial A \to \partial A$ satisfying

$$\partial U_0 = 1_{\partial A}, \quad \partial U_{t+s} = (\partial U_t)(\partial U_s), \quad (\partial U_t)^\dagger = \partial U_{-t}. \tag{15}$$

For every boundary spectral projector P_x,

$$\mathsf{T}_{U_t}(P_x) = (\partial U_t) P_x (\partial U_t)^\dagger. \tag{16}$$

HILBERT REALIZATION AND BOUNDARY STONE THEOREM

The next result is deliberately representation-dependent. The claim is not that a Hamiltonian exists internally in every abstract dagger category. Rather, once a finite boundary spectral process category is faithfully realized in finite-dimensional Hilbert spaces, reversible boundary spectral flows reconstruct ordinary Hamiltonian dynamics.

Definition 7 (Faithful boundary Hilbert realization)**.** *A faithful boundary Hilbert realization of $\mathbb{B}$ is a dagger symmetric monoidal functor $F : \mathcal{C} \to$ **FdHilb** such that: $F(I) = \mathbb{C}$; F preserves daggers, tensor products, and finite dagger biproducts; $F(P)$ is an orthogonal projection whenever P is a boundary spectral projector; F is faithful on boundary projectors; and for each reversible boundary spectral flow U_t, the map $t \mapsto F(\partial U_t)$ is norm-continuous.*

Theorem 2 (Finite-dimensional boundary Stone theorem)**.** *Let $\mathbb{B}$ be a boundary spectral process category with a faithful boundary Hilbert realization F. Let $U_t : A \to A$ be a reversible boundary spectral flow. Then there exists a unique self-adjoint operator*

$$H_{\partial A} : F(\partial A) \to F(\partial A) \tag{17}$$

such that

$$F(\partial U_t) = e^{-itH_{\partial A}} \tag{18}$$

for all $t \in \mathbb{R}$.

Proof. Define $V_t := F(\partial U_t)$. Functoriality gives

$$V_{t+s} = V_t V_s, \qquad V_0 = 1_{F(\partial A)}. \tag{19}$$

Since $(\partial U_t)^\dagger = \partial U_{-t}$ and F preserves daggers,

$$V_t^\dagger = F((\partial U_t)^\dagger) = F(\partial U_{-t}) = V_{-t}. \tag{20}$$

Hence $V_t V_t^\dagger = V_t V_{-t} = V_0 = 1$, and similarly $V_t^\dagger V_t = 1$. Thus each V_t is unitary.

By assumption, $t \mapsto V_t$ is norm-continuous. Since $F(\partial A)$ is finite-dimensional, V_t is a norm-continuous one-parameter unitary group. Choosing an orthonormal basis identifies this group with a continuous homomorphism $\mathbb{R} \to U(n)$, where $n = \dim F(\partial A)$. Every continuous one-parameter subgroup of $U(n)$ is of the form $V_t = e^{tK}$ for a unique $K \in \mathfrak{u}(n)$, the Lie algebra of skew-adjoint matrices. Thus $K^\dagger = -K$.

Set $H_{\partial A} := iK$. Then

$$H_{\partial A}^\dagger = (iK)^\dagger = (-i)K^\dagger = (-i)(-K) = iK = H_{\partial A}. \tag{21}$$

Moreover, $K = -iH_{\partial A}$, so

$$V_t = e^{tK} = e^{-itH_{\partial A}}. \tag{22}$$

This proves existence.

If $H'_{\partial A}$ is another self-adjoint operator satisfying $e^{-itH_{\partial A}} = e^{-itH'_{\partial A}}$ for all t, differentiating at $t = 0$ yields $-iH_{\partial A} = -iH'_{\partial A}$, hence $H_{\partial A} = H'_{\partial A}$. This proves uniqueness. □

SPECTRAL RECONSTRUCTION ON THE BOUNDARY

Theorem 3 (Boundary spectral reconstruction)**.** *Let $\mathbb{B}$ be a boundary spectral process category with faithful boundary Hilbert realization F. Let $U_t : A \to A$ be a reversible boundary spectral flow. Suppose the generator $H_{\partial A}$ from the preceding theorem has finite spectral decomposition*

$$H_{\partial A} = \sum_{\lambda \in \Lambda} \lambda E_\lambda \tag{23}$$

and each E_λ lies in the image of a unique boundary spectral projector $P_\lambda : \partial A \to \partial A$. Then the boundary flow is reconstructed as

$$F(\partial U_t) = \sum_{\lambda \in \Lambda} e^{-it\lambda} E_\lambda, \tag{24}$$

the boundary spectral observable $O_H = (\Lambda, \{P_\lambda\}_{\lambda \in \Lambda})$ is uniquely determined, and for every boundary-normalized state $\psi : I \to A$,

$$p_\psi^{O_H}(\lambda) = \langle \rho_A \psi, E_\lambda \rho_A \psi \rangle \tag{25}$$

under the Hilbert realization.

Proof. The boundary Stone theorem gives $F(\partial U_t) = e^{-itH_{\partial A}}$. Since $H_{\partial A}$ is finite-dimensional and self-adjoint, the spectral theorem gives mutually orthogonal projections E_λ summing to the identity and satisfying $H_{\partial A} = \sum_{\lambda \in \Lambda} \lambda E_\lambda$. Finite-dimensional functional calculus gives

$$e^{-itH_{\partial A}} = \sum_{\lambda \in \Lambda} e^{-it\lambda} E_\lambda. \tag{26}$$

By hypothesis, each E_λ has a unique representing boundary spectral projector P_λ, so $O_H = (\Lambda, \{P_\lambda\})$ is uniquely determined. For a boundary-normalized state ψ,

$$\begin{aligned} F(p_\psi^{O_H}(\lambda)) &= F((\rho_A\psi)^\dagger P_\lambda (\rho_A \psi)) && (27) \\ &= F(\rho_A\psi)^\dagger F(P_\lambda) F(\rho_A\psi) && (28) \\ &= F(\rho_A\psi)^\dagger E_\lambda F(\rho_A\psi), && (29) \end{aligned}$$

which is the displayed Hilbert-space probability. □

GAUGE REDUNDANCY AND PHYSICAL SYMMETRY

Definition 8 (Boundary-gauge transformation)**.** *A reversible causal automorphism $r : A \to A$ is a boundary-gauge transformation if its induced boundary unitary is the identity:*

$$\partial r = 1_{\partial A}. \tag{30}$$

Equivalently, for every boundary spectral projector $P : \partial A \to \partial A$, $\mathsf{T}_r(P) = P$.

Theorem 4 (Gauge invisibility)**.** *Let $r : A \to A$ be a boundary-gauge transformation. Then for every boundary spectral observable $O = (X, \{P_x\}_{x\in X})$, every boundary-normalized state $\psi : I \to A$, and every $x \in X$,*

$$p^O_{r\psi}(x) = p^O_\psi(x). \tag{31}$$

Proof. Since r is boundary-gauge, $\partial r = 1_{\partial A}$. Boundary compatibility gives $\rho_A r = (\partial r)\rho_A = \rho_A$. Therefore

$$p^O_{r\psi}(x) = (\rho_A r\psi)^\dagger P_x(\rho_A r\psi) \tag{32}$$
$$= (\rho_A\psi)^\dagger P_x(\rho_A\psi) = p^O_\psi(x). \tag{33}$$

□

Definition 9 (Boundary physical symmetry)**.** *A reversible causal automorphism $s : A \to A$ is a boundary physical symmetry if ∂s is nontrivial but preserves the family of admissible boundary spectral observables. For $O = (X, \{P_x\})$, define $s \cdot O = (X, \{(\partial s)P_x(\partial s)^\dagger\})$.*

Theorem 5 (Covariance of physical symmetries)**.** *Let $s : A \to A$ be a boundary physical symmetry. Then, for every boundary-normalized $\psi : I \to A$,*

$$p^{s\cdot O}_{s\psi}(x) = p^O_\psi(x). \tag{34}$$

Proof. Let $Q_x = (\partial s)P_x(\partial s)^\dagger$. By boundary compatibility, $\rho_A s = (\partial s)\rho_A$. Hence

$$p^{s\cdot O}_{s\psi}(x) = ((\partial s)\rho_A\psi)^\dagger Q_x((\partial s)\rho_A\psi) \tag{35}$$
$$= (\rho_A\psi)^\dagger(\partial s)^\dagger Q_x(\partial s)\rho_A\psi. \tag{36}$$

Since ∂s is unitary, $(\partial s)^\dagger Q_x(\partial s) = P_x$. Therefore $p^{s\cdot O}_{s\psi}(x) = p^O_\psi(x)$. □

BOUNDARY ERROR CORRECTION

Quantum error correction protects logical information against physical errors. In the present framework, logical information is encoded as boundary spectral data.

Definition 10 (Boundary code)**.** *A boundary code consists of a logical object L, a physical object A, an encoding morphism $e : L \to A$, and a boundary identification $\partial A \simeq L$. For simplicity, assume this identification is strict, so $\partial A = L$, and let $\rho_A : A \to L$ be the logical boundary restriction.*

Definition 11 (Boundary-correctable error family)**.** *Let $\mathsf{Err}(A)$ be a family of causal endomorphisms of A. The code is boundary-correctable for $\mathsf{Err}(A)$ if for each $E \in \mathsf{Err}(A)$, there exists a recovery morphism $R_E : A \to A$ such that*

$$\rho_A R_E E e = 1_L. \tag{37}$$

Theorem 6 (Preservation of logical spectral statistics)**.** *Let $e : L \to A$ be a boundary code correctable for $\mathsf{Err}(A)$. Let $E \in \mathsf{Err}(A)$ and let R_E satisfy $\rho_A R_E E e = 1_L$. Then for every logical boundary spectral observable $O = (X, \{P_x : L \to L\}_{x\in X})$ and every normalized logical state $\psi : I \to L$,*

$$p^O_{R_E E e\psi}(x) = p^O_\psi(x). \tag{38}$$

Proof. By definition,

$$p^O_{R_E E e\psi}(x) = (\rho_A R_E E e\psi)^\dagger P_x(\rho_A R_E E e\psi). \tag{39}$$

Using $\rho_A R_E E e = 1_L$, we obtain $\rho_A R_E E e\psi = \psi$. Therefore $p^O_{R_E E e\psi}(x) = \psi^\dagger P_x\psi = p^O_\psi(x)$. □

The standard Knill-Laflamme condition states that a code projector P_C corrects an error set $\{E_a\}$ precisely when

$$P_C E_a^\dagger E_b P_C = \alpha_{ab} P_C \tag{40}$$

for some matrix (α_{ab}) [7]. In the present language this condition implies that, after syndrome extraction and recovery, the logical boundary restriction is restored. Thus correctability is precisely preservation of all logical boundary spectral statistics.

EXAMPLE: TORIC CODE AS BOUNDARY SPECTRAL PROTECTION

Consider the toric code on a square lattice embedded on a torus [8]. Physical qubits live on edges. The stabilizer group is generated by star operators

$$A_v = \prod_{e\ni v} X_e \tag{41}$$

and plaquette operators

$$B_p = \prod_{e\in\partial p} Z_e. \tag{42}$$

The code space C is the simultaneous $+1$ eigenspace of all stabilizers. Logical operators are represented by noncontractible loop operators $\overline{Z}_1, \overline{Z}_2, \overline{X}_1, \overline{X}_2$, which commute with all stabilizers but are not products of stabilizers.

Let A be the full physical system and let L be the logical two-qubit system. Define $\partial A = L$ and let $\rho_A : A \to L$ be the ideal decoding map that extracts the logical degrees of freedom from the code space after syndrome correction. Boundary spectral observables are projective measurements of logical operators. For example, the logical observable $\overline{Z}_1$ has projectors

$$P_+^{\overline{Z}_1} = \frac{1 + \overline{Z}_1}{2}, \qquad P_-^{\overline{Z}_1} = \frac{1 - \overline{Z}_1}{2}. \tag{43}$$

Let E be a Pauli error supported on fewer than d edges, where d is the code distance. If E is correctable, there exists a recovery R_E such that $\rho_A R_E E e = 1_L$. Hence, by the preservation theorem,

$$p^O_{R_E E e\psi}(x) = p^O_\psi(x) \tag{44}$$

for every logical spectral observable O. Thus local errors below the code distance are invisible to logical boundary spectral statistics after recovery.

If E is a Pauli string along a noncontractible cycle, then E acts as a logical operator. In that case $\rho_A E e \neq 1_L$; for example, $\rho_A E e = \overline{X}_1$. Such an error is not boundary-gauge. It is a physical boundary symmetry or logical fault, transforming boundary spectral observables by conjugation. This gives a categorical distinction between local correctable errors, which are boundary-gauge after recovery, and noncontractible logical errors, which act nontrivially on boundary spectral data.

DISCUSSION AND CONCLUSION

The theory developed here is intentionally modest in one respect and ambitious in another. It is modest because it does not claim to derive Hilbert space from purely categorical assumptions. The Hamiltonian reconstruction result depends on a faithful finite-dimensional Hilbert realization. In this sense, the Stone theorem proved above is a boundary representation theorem, not a representation-free categorical Stone theorem.

It is ambitious because it identifies a structural layer often left implicit: physical observability may be boundary-mediated, and the transport of spectral projectors is more fundamental operationally than the transport of abstract observable labels. This viewpoint is especially natural for topological phases, gauge theories, quantum error correction, and open quantum systems in which coarse-grained boundary observables remain stable under environmental coupling.

The formalism suggests a useful hierarchy:

$$\begin{aligned}\text{bulk morphism} &\to \text{boundary morphism}\\ &\to \text{projector transport}\\ &\to \text{observable statistics.}\end{aligned} \tag{45}$$

Gauge redundancy is triviality at the boundary-morphism level. Physical symmetry is nontrivial but covariant boundary action. Logical failure in a code is nontrivial action on protected boundary spectral projectors.

We introduced boundary spectral process categories as a finite-dimensional categorical framework for boundary-sensitive quantum dynamics. The central addition to standard process-theoretic quantum mechanics is explicit spectral projector transport on boundary systems. The framework recovers ordinary finite-dimensional quantum mechanics under faithful Hilbert realization, while giving a compositional language for boundary observability, gauge redundancy, and protected logical structure.

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