Multi-Stage Propulsion System with Variable-Geometry Corrugate Lobe Exhaust Architecture for Enhanced Supersonic Thrust Vectoring and Mixing Efficiency

Yu Murakami, New York General Group
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Technical Field

The present invention relates to aerospace propulsion systems, and more particularly to an advanced rocket propulsion assembly incorporating variable-geometry corrugated lobe nozzle configurations optimized for supersonic exhaust flow management, enhanced propellant mixing efficiency, and improved thrust performance across multiple atmospheric flight regimes.

Background of the Invention

Contemporary rocket propulsion systems face persistent challenges in optimizing exhaust flow characteristics during supersonic and hypersonic flight conditions. Conventional convergent-divergent nozzle geometries, while effective for basic thrust generation, exhibit inherent limitations in managing shock wave structures, boundary layer separation phenomena, and downstream mixing processes that significantly impact overall propulsion efficiency and vehicle controllability.

Traditional conical and bell-shaped nozzle configurations produce relatively uniform exhaust plumes that, while predictable, fail to capitalize on potential aerodynamic advantages offered by more complex geometrical arrangements. The interaction between supersonic exhaust gases and ambient atmospheric conditions creates shock wave patterns that dissipate significant energy through thermal and acoustic mechanisms, reducing net propulsive efficiency and contributing to undesirable acoustic signatures that complicate vehicle integration and ground support operations.

The phenomenon of flow separation within divergent nozzle sections represents a particularly problematic aspect of conventional designs. When operating at nozzle pressure ratios below design conditions, the boundary layer detaches from nozzle walls, creating recirculation zones that reduce effective thrust area and introduce asymmetric force components that compromise vehicle stability. Existing mitigation approaches, including altitude-compensating nozzle designs and mechanical insertion devices, add substantial mass, complexity, and potential failure modes to propulsion systems.

Furthermore, the mixing characteristics of exhaust gases with ambient air or secondary propellant streams remain suboptimal in traditional configurations. Efficient mixing is essential for numerous applications including afterburning systems, air-augmented rocket propulsion, and thermal management of downstream vehicle structures. The shear layer development in conventional round or conical nozzles proceeds at rates insufficient for many advanced propulsion concepts, necessitating extended mixing lengths that impose vehicle geometry constraints and performance penalties.

Lobed mixer nozzle technology, originally developed for turbofan engine applications, demonstrates significant potential for enhancing mixing rates through the generation of streamwise vortical structures. However, direct application of turbofan lobed mixer principles to rocket propulsion systems operating at substantially higher pressure ratios and temperature differentials requires fundamental reconceptualization of the underlying flow physics and geometric optimization parameters.

Previous attempts to incorporate lobed structures into convergent-divergent nozzle architectures have encountered difficulties in managing the complex shock wave interactions that arise when supersonic flow passes through non-axisymmetric passage geometries. The three-dimensional nature of shock structures in lobed nozzle configurations creates regions of locally intense heating and pressure loading that challenge structural integrity and thermal protection system design.

The relationship between corrugation extent within the divergent section and resulting flow behavior has not been systematically exploited in practical propulsion system designs. While fundamental research indicates that the longitudinal distribution of lobe amplitude profoundly influences shock wave coalescence, mixing layer development, and separation onset characteristics, translation of these insights into implementable propulsion hardware remains incomplete.

There exists therefore a significant need for a propulsion system architecture that systematically leverages corrugated lobe geometry principles to achieve superior performance across the multiple operational domains encountered during rocket-powered flight, while maintaining the reliability, manufacturability, and structural robustness required for practical aerospace applications.

Summary of the Invention

The present invention provides a multi-stage rocket propulsion system incorporating an innovative exhaust nozzle architecture characterized by precisely engineered corrugated lobe geometry extending through a controlled portion of the divergent section. The system achieves unprecedented combinations of mixing efficiency, shock wave management, and separation resistance through optimization of the corrugation extent at approximately sixty percent of the divergent section length, representing a fundamental departure from both conventional smooth-wall nozzles and fully-corrugated lobe mixer designs.

The propulsion system of the present invention comprises a primary combustion chamber assembly, a throat section of circular cross-section, a convergent inlet section of conventional geometry, and a divergent exhaust section featuring a plurality of circumferentially-distributed lobes with corrugation initiating at a predetermined axial station downstream of the throat plane and extending to the nozzle exit plane. The corrugated portion encompasses approximately sixty percent of the total divergent section length, with the upstream forty percent maintaining smooth axisymmetric geometry that establishes stable supersonic flow prior to introduction of three-dimensional perturbations.

The lobe architecture generates a characteristic "double-diamond" shock pattern within the corrugated region that progressively coalesces into a single-diamond configuration in the downstream exhaust plume. This controlled shock coalescence process extracts energy from undesirable shock oscillations while preserving the streamwise vortical structures essential for rapid mixing. The spatial evolution of shock geometry from complex multi-element patterns to simpler unified structures reduces acoustic emission intensity and minimizes the potential for resonant coupling between shock oscillations and vehicle structural modes.

The sixty-percent corrugation extent represents an optimal balance between competing physical mechanisms that govern supersonic lobed nozzle performance. Corrugation lengths substantially exceeding this value produce excessive dissipation within the nozzle, reducing exit momentum and introducing thermal loading challenges in the extended corrugated section. Conversely, corrugation lengths significantly below this value necessitate more abrupt lobe geometry to achieve equivalent exit plane configuration, resulting in divergence angles that exceed critical values for boundary layer attachment and precipitate detrimental flow separation.

The propulsion system incorporates structural provisions for the unique loading patterns generated by three-dimensional supersonic flow through the corrugated lobe section. The lobe peaks experience compression loading from the adjacent flow expansion regions, while the lobe troughs sustain tensile stress concentrations from the converging flow paths. The wall thickness distribution varies circumferentially and axially to accommodate these non-uniform loading conditions while minimizing overall nozzle mass.

Thermal management provisions address the concentrated heating that occurs at lobe peak leading edges where shock wave impingement creates localized stagnation conditions. Regenerative cooling channel geometry follows the three-dimensional lobe contours, with increased coolant flow velocity through peak regions achieved through controlled channel cross-sectional area reduction. Alternative embodiments incorporate ablative thermal protection in the corrugated section for applications where regenerative cooling complexity is undesirable.

The propulsion system optionally incorporates active geometry modification capability that enables adjustment of the effective corrugation extent during flight. Sliding sleeve elements within the divergent section can extend or retract to expose varying portions of the underlying corrugated lobe structure, permitting optimization of nozzle characteristics for different atmospheric pressure and flight velocity conditions. The actuation system utilizes pressure-balanced mechanisms that minimize force requirements while providing deterministic positioning across the operational envelope.

A further aspect of the invention provides for staged propulsion configurations wherein multiple nozzle assemblies with differing corrugation extents operate in sequence or in combination. First-stage applications emphasize extended corrugation for maximum sea-level mixing enhancement, while upper-stage applications employ reduced corrugation extent optimized for vacuum operation where ambient interaction effects are negligible. Parallel-burning configurations may incorporate nozzles of varying corrugation extent to provide throttling-like control of aggregate mixing characteristics without mechanical throttling of individual engines.

The manufacturing methodology for the corrugated lobe nozzle section accommodates the complex three-dimensional geometry through advanced additive manufacturing processes that build the complete divergent section as a monolithic component. The layer-by-layer construction approach inherently supports the varying wall thickness requirements and integrated cooling channel geometries that would present significant challenges for conventional subtractive or forming-based manufacturing methods.

Quality assurance provisions address the unique inspection challenges posed by the internal corrugated geometry. Computed tomographic scanning verifies dimensional conformance of the complex lobe profiles, while specialized ultrasonic transducer arrays confirm wall thickness and detect internal defects in regions inaccessible to conventional inspection techniques. Surface finish requirements for the corrugated section balance aerodynamic considerations against practical manufacturing capabilities, with controlled surface roughness

specifications that acknowledge the reduced sensitivity of mixing-dominated flow regimes to small-scale wall perturbations.

Detailed Description of the Invention

The present invention constitutes a comprehensive propulsion system architecture that systematically exploits the relationship between corrugated lobe geometry and supersonic flow behavior to achieve unprecedented performance across multiple operational domains. The development of this system draws upon fundamental principles of compressible fluid dynamics, three-dimensional shock wave theory, and vortex-dominated mixing processes, synthesizing these elements into a unified propulsion concept that addresses longstanding limitations of conventional rocket nozzle designs.

The core innovation resides in the recognition that the longitudinal extent of corrugation within a convergent-divergent nozzle profoundly influences the resulting flow field characteristics in ways that can be systematically optimized for specific performance objectives. Through extensive analytical investigation of the relationships between geometric parameters and flow behavior, the present invention identifies the corrugation extent of approximately sixty percent of the divergent section length as representing an optimal configuration that balances competing physical mechanisms governing supersonic lobed nozzle performance.

The propulsion system operates across the complete range of atmospheric conditions encountered during rocket-powered flight, from sea-level launch through upper atmospheric ascent to vacuum orbital insertion. The corrugated lobe architecture provides inherent adaptability to varying ambient pressure conditions while maintaining the structural robustness and thermal management capability essential for reliable operation in the demanding environment of rocket propulsion.

The following detailed description presents the complete technical disclosure necessary for implementation of the invention by those skilled in the relevant arts. The description proceeds from fundamental geometric specifications through flow field behavior, structural considerations, thermal management approaches, manufacturing processes, and operational procedures, providing comprehensive guidance for practical realization of the disclosed propulsion system.

The geometric configuration of the propulsion system is most precisely described with reference to a cylindrical coordinate system having its origin at the centroid of the throat plane, with the axial coordinate z directed downstream along the nozzle axis, the radial coordinate r extending outward from the axis, and the azimuthal coordinate θ measured from a reference meridian plane. All geometric specifications in the following description employ this coordinate system unless otherwise noted.

The throat section occupies the region from z equals negative five millimeters to z equals positive five millimeters for a representative implementation having throat diameter of one hundred twenty millimeters. The throat contour follows a circular arc profile in the meridional plane with radius of curvature equal to two point five times the throat radius, providing smooth acceleration through the sonic condition while avoiding the sharp geometric transitions that would introduce undesirable wave patterns in the transonic region.

The convergent inlet section extends from z equals negative one hundred thirty millimeters to z equals negative five millimeters, with the upstream boundary defined by the combustion chamber interface plane. The convergent section wall radius varies according to a fifth-order polynomial function of axial position, with coefficients selected to provide zero wall slope and zero wall curvature at both the chamber interface and the throat approach stations. This polynomial formulation, known in the technical literature as a Rao contour after the pioneering work of G.V.R. Rao at the Jet Propulsion Laboratory, minimizes boundary layer momentum thickness at the throat while maintaining attached flow throughout the convergent section.

The divergent exhaust section extends from z equals positive five millimeters to z equals positive four hundred twenty millimeters, providing a total divergent length of four hundred fifteen millimeters. This divergent section length establishes an overall length-to-throat-diameter ratio of approximately three point five, representing a moderate value that balances performance considerations against envelope and mass constraints.

The divergent section divides into two distinct regions characterized by fundamentally different geometric philosophies. The upstream smooth portion extends from z equals positive five millimeters to z equals positive one hundred seventy-one millimeters, occupying forty percent of the total divergent length. Throughout this region, the wall geometry maintains axisymmetric character with radius varying according to a prescribed expansion contour optimized for efficient supersonic acceleration.

The wall radius within the upstream smooth portion follows a modified Rao bell contour that provides initial rapid expansion followed by progressive reduction in expansion rate as the flow approaches the corrugation initiation station. At z equals positive five millimeters, the wall radius equals sixty millimeters corresponding to the throat half-diameter. At z equals positive one hundred seventy-one millimeters, the wall radius has increased to one hundred fourteen millimeters, corresponding to an area ratio of approximately three point six relative to the throat. The local wall angle at the corrugation initiation station equals eight point three degrees relative to the nozzle axis, selected to be sufficiently gradual that the boundary layer approaching the corrugated section possesses adequate momentum to resist separation when subsequently encountering the three-dimensional lobe geometry.

The downstream corrugated lobe portion extends from z equals positive one hundred seventy-one millimeters to z equals positive four hundred twenty millimeters, occupying sixty percent of the total divergent length. Within this region, the wall departs from axisymmetric geometry to assume the three-dimensional corrugated character that constitutes the distinctive feature of the present invention.

The corrugated lobe geometry is defined by superposition of a circumferentially-varying amplitude function upon the baseline axisymmetric expansion contour. The local wall radius at any point within the corrugated section is expressed as the sum of a mean radius component that varies only with axial position and a perturbation component that varies with both axial position and azimuthal angle.

For the representative implementation incorporating eight lobes, the perturbation component varies as a cosine function of eight times the azimuthal angle, with amplitude increasing from zero at the corrugation initiation station to maximum value at the exit plane. The amplitude at the exit plane equals eighteen millimeters, representing approximately eleven percent of the mean exit radius. This amplitude magnitude generates pressure perturbations of sufficient strength to drive vigorous vortical mixing while remaining within limits that maintain attached boundary layer flow.

The mean radius at the exit plane equals one hundred sixty-two millimeters, corresponding to an exit area ratio of seven point three relative to the throat area. This expansion ratio is appropriate for the design nozzle pressure ratio of fifty, typical of first-stage rocket engines operating with kerosene and liquid oxygen propellants at chamber pressures of approximately seventy bar.

The amplitude growth function within the corrugated section follows a modified cosine distribution rather than simple linear variation. The amplitude equals zero at the corrugation initiation station, increases slowly through the initial portion of the corrugated section, accelerates through the middle portion, and approaches the maximum value asymptotically near the exit plane. This amplitude distribution provides gradual introduction of the three-dimensional perturbation when the flow first encounters the corrugated geometry, avoiding the abrupt wall angle changes that would otherwise occur if linear amplitude growth were employed with the required total amplitude change.

The specific amplitude distribution function incorporates a single adjustable parameter that controls the rate of amplitude growth. For the representative implementation, this parameter is selected such that fifty percent of the total amplitude is achieved at a point seventy percent of the distance from the corrugation initiation station to the exit plane. This relatively delayed amplitude growth biases the geometric change toward the downstream portion of the corrugated section where the expanded flow possesses reduced static pressure and correspondingly reduced tendency toward separation.

The transition between peak and trough regions around the lobe circumference follows a smoothly-varying profile that avoids the sharp corners that would create localized flow separation and attendant performance degradation. Each lobe spans forty-five degrees of azimuthal arc for the eight-lobe configuration, with the peak centered at angles of zero, forty-five, ninety, one hundred thirty-five, one hundred eighty, two hundred twenty-five, two hundred seventy, and three hundred fifteen degrees from the reference meridian. The trough regions center at angles offset by twenty-two point five degrees from the peak locations.

The transition region between peak and trough spans approximately fifteen degrees of azimuthal arc on each side of the peak and trough center lines. Within this transition region, the radial position varies according to a raised-cosine blending function that provides continuous first and second derivatives of the wall surface, eliminating the curvature discontinuities that would otherwise generate undesirable wave patterns and structural stress concentrations.

The wall thickness distribution throughout the nozzle accommodates the varying structural and thermal requirements encountered at different locations. Within the throat region, where heat flux reaches maximum values due to the combination of high gas temperature and high convective heat transfer coefficient associated with the sonic flow condition, wall thickness equals four point five millimeters with regenerative cooling channels occupying an additional three millimeters of radial extent. Within the convergent section, wall thickness decreases progressively upstream to three point two millimeters at the chamber interface, reflecting the reduced thermal loading in this region.

Within the upstream smooth portion of the divergent section, wall thickness decreases from four point two millimeters immediately downstream of the throat to three point zero millimeters at the corrugation initiation station, following the axial variation in thermal loading that results from flow expansion and consequent reduction in gas temperature and density. Within the corrugated section, wall thickness varies both axially and circumferentially to address the three-dimensional loading patterns characteristic of this region.

At lobe peak locations within the corrugated section, wall thickness equals three point two millimeters at the corrugation initiation station and increases progressively to four point zero millimeters at the exit plane. This thickness increase toward the exit reflects the increasing structural loading associated with the growing lobe amplitude and the concentrated shock impingement that occurs at peak leading edges. At lobe trough locations, wall thickness exceeds the peak

values by approximately twenty percent at corresponding axial stations, providing additional material to resist the elevated hoop stress that occurs where the local radius of curvature falls below the mean value.

The transition between peak and trough thickness values follows the same azimuthal distribution function as the wall radius variation, ensuring consistent structural behavior around the lobe circumference. The thickness variation is implemented through corresponding adjustment of the external wall surface while maintaining the precisely-defined internal flow surface geometry.

The flow physics within the upstream smooth portion of the divergent section follows the well-established principles of quasi-one-dimensional supersonic expansion modified by the three-dimensional effects associated with the actual nozzle geometry. Understanding the flow state at the corrugation initiation station is essential for predicting subsequent behavior within the corrugated section.

Combustion products entering the throat section from the convergent inlet possess static temperature of approximately three thousand five hundred Kelvin, static pressure of approximately thirty-nine bar (corresponding to the critical pressure ratio of approximately one point eight for the combustion gas specific heat ratio of one point two four), and velocity equal to the local speed of sound of approximately one thousand one hundred meters per second. The flow Mach number equals unity throughout the throat plane, with small spatial variations associated with boundary layer displacement effects near the walls.

As the flow expands through the upstream smooth portion of the divergent section, it accelerates continuously while static pressure and temperature decrease in accordance with the isentropic expansion relations. At the corrugation initiation station located at z equals positive one hundred seventy-one millimeters, the flow has achieved Mach number of approximately two point one, static pressure has decreased to approximately five point three bar, and static temperature has decreased to approximately two thousand two hundred Kelvin.

The boundary layer at the corrugation initiation station has developed through the combined influences of the favorable pressure gradient in the convergent section, the adverse pressure gradient in the subsonic portion of the throat region, and the favorable pressure gradient of the supersonic expansion. For the turbulent boundary layer conditions characteristic of rocket nozzle Reynolds numbers (throat Reynolds number of approximately fifteen million based on throat diameter and sonic conditions), the boundary layer momentum thickness at the corrugation initiation station equals approximately zero point three five millimeters.

The displacement thickness, representing the effective outward displacement of the inviscid flow due to the velocity deficit within the boundary layer, equals approximately zero point nine millimeters at the corrugation initiation station. This displacement thickness is incorporated into the geometric design through corresponding reduction of the physical wall radius relative to the nominal inviscid flow boundary, ensuring that the effective area ratio for the inviscid core flow matches the design intent.

The shape factor, defined as the ratio of displacement thickness to momentum thickness, equals approximately two point six at the corrugation initiation station. This value falls within the range characteristic of attached turbulent boundary layers under moderate favorable pressure gradient conditions, confirming that the boundary layer entering the corrugated section possesses robust attached character with substantial resistance to separation.

The velocity profile within the boundary layer follows the law-of-the-wall formulation appropriate for high-speed compressible turbulent flow. In the immediate vicinity of the wall, the velocity increases linearly with distance from the wall surface according to the viscous sublayer relationship. At greater distances, the velocity profile transitions through a buffer region to the logarithmic overlap layer where velocity varies as the natural logarithm of wall distance. At still greater distances, the velocity profile departs from the logarithmic relationship and blends smoothly into the freestream velocity at the boundary layer edge.

The turbulence intensity at the boundary layer edge equals approximately two point five percent at the corrugation initiation station, representing the residual fluctuation level that persists from the combustion chamber flow and subsequent acceleration through the nozzle. This turbulence level influences the subsequent development of mixing layers at the interface between the exhaust jet and ambient fluid, with higher turbulence promoting more rapid mixing but also potentially contributing to flow instabilities under certain conditions.

The static temperature at the wall surface within the upstream smooth section is governed by the balance between convective heat transfer from the hot combustion gases and heat removal by the regenerative cooling system. For the representative implementation with hydrocarbon fuel regenerative cooling, wall temperature varies from approximately nine hundred Kelvin near the throat, where heat flux is maximum, to approximately seven hundred Kelvin at the corrugation initiation station, where the reduced gas temperature and density diminish convective loading.

The flow entering the corrugated section encounters progressively increasing geometric perturbation as it advances downstream, generating the three-dimensional flow patterns that constitute the essential mechanism of the present invention. The physics governing this three-dimensional flow development encompasses multiple interacting phenomena including supersonic wave patterns, secondary flow generation, vortex development, and boundary layer response to three-dimensional pressure gradients.

As the supersonic flow approaches a lobe peak, it encounters a wall contour that deflects the flow radially inward relative to the trajectory it would follow along the mean axisymmetric expansion surface. This inward deflection constitutes a compressive disturbance that generates an oblique shock wave emanating from the wall and propagating inward toward the nozzle axis. The shock angle relative to the incoming flow direction depends upon the local Mach number and the magnitude of the wall deflection, with stronger deflections producing shock waves at greater angles to the flow direction.

For the amplitude distribution of the representative implementation, the wall deflection at lobe peaks increases progressively from zero at the corrugation initiation station to approximately five degrees at the exit plane. At the local Mach number of two point one prevailing at the corrugation initiation station, even small deflection angles generate weak oblique shock waves that propagate inward at angles of approximately fifty-five degrees relative to the flow direction. At the higher Mach numbers prevailing near the exit (approximately Mach two point seven for the representative area ratio), the shock angles decrease to approximately forty degrees for the same deflection magnitude.

Simultaneously with the compression at lobe peaks, the flow adjacent to lobe troughs encounters a wall contour that deflects the flow radially outward, creating an expansion process that reduces local pressure and increases local velocity. This expansion occurs through a continuous Prandtl-Meyer fan rather than through a discrete wave discontinuity, with the fan angle span depending upon the total turning angle and local Mach number.

The juxtaposition of compression at peaks and expansion at troughs creates a circumferentially-varying pressure distribution that constitutes the primary driver of secondary flow within the corrugated section. The pressure at lobe peaks exceeds the pressure at lobe troughs by an amount that increases with the local lobe amplitude, reaching values on the order of one point five bar at the exit plane for the representative implementation. This pressure differential creates a circumferential pressure gradient that accelerates fluid from peak regions toward trough regions along paths in the circumferential-radial plane perpendicular to the primary axial flow direction.

The secondary flow driven by the circumferential pressure gradient rolls up into streamwise-oriented vortical structures that wrap helically around the nozzle axis as they convect downstream with the primary flow. Each lobe peak generates a counter-rotating vortex pair with the two vortices positioned symmetrically on opposite sides of the peak centerline. The vortex on the clockwise side of the peak (when viewed from downstream) rotates in the counter-clockwise direction, while the vortex on the counter-clockwise side rotates clockwise.

The strength of the generated vortices, measured by circulation magnitude, increases with both the local pressure differential and the axial extent over which the pressure gradient acts. For the progressive amplitude growth characteristic of the present invention, vortex strength builds continuously through the corrugated section rather than achieving full strength at a single initiation station. This distributed vortex development creates more robust vortical structures with greater resistance to disruption by turbulent fluctuations or adverse pressure gradients.

The circulation of individual vortices at the exit plane reaches values of approximately twelve square meters per second for the representative implementation. This circulation magnitude creates tangential velocities within the vortex cores of approximately one hundred fifty meters per second, sufficient to substantially distort the circular cross-section of the exhaust jet into the lobed pattern characteristic of mixer nozzles.

The oblique shock waves generated at individual lobe peaks propagate inward across the flow field, eventually encountering shock waves generated at other lobe peaks and interacting through the wave intersection processes described by classical supersonic flow theory. For the eight-lobe configuration, eight shock waves propagate inward from the eight peak locations, creating a complex pattern of shock intersections, reflections, and transmitted waves.

The shock intersection pattern visible in cross-sectional planes perpendicular to the nozzle axis exhibits the characteristic double-diamond structure referenced in the prior art literature. At stations near the corrugation initiation where lobe amplitudes remain small, the eight individual shock waves are distinguishable as separate features, though closely spaced. Progressing downstream, the shock waves strengthen as lobe amplitudes increase, and the shock intersection points move radially inward as the accumulated flow deflection increases.

At an axial station approximately midway through the corrugated section, the shock intersection geometry achieves the fully-developed double-diamond pattern comprising eight primary shock segments extending from the lobe peaks toward the axis, eight reflected shock segments propagating outward from the intersection points, and a central region of complex wave interactions near the axis. This double-diamond pattern is most clearly visible in schlieren photographs or computational visualizations of the density gradient field.

As the flow continues downstream and exits the nozzle, the shock pattern undergoes progressive coalescence into simpler unified structures. The eight individual peak-generated shocks and their associated reflections merge through mutual interaction, eventually forming a single approximately conical shock surface surrounding the exhaust jet. This single-diamond pattern persists into the

downstream plume, gradually weakening through expansion and viscous dissipation until the shock structure eventually disappears entirely at sufficient distance downstream.

The coalescence process from double-diamond to single-diamond pattern occurs over an axial distance of approximately two to four exit diameters downstream of the exit plane, depending upon the nozzle pressure ratio and ambient conditions. At nozzle pressure ratios near the design value of fifty, the coalescence occurs relatively rapidly due to the strong shock waves and their vigorous interaction. At lower nozzle pressure ratios corresponding to lower-altitude operation, weaker shock waves coalesce more gradually.

The energy dissipated through the shock coalescence process represents a combination of useful conversion from small-scale oscillatory motion to organized vortical motion and undesirable conversion to thermal energy through irreversible shock heating. The sixty-percent corrugation extent of the present invention is specifically selected to achieve beneficial shock pattern development and vortex generation while minimizing the internal shock dissipation losses that would accumulate with more extended corrugation.

The boundary layer within the corrugated section experiences three-dimensional pressure gradients, streamline curvature, and centrifugal effects that profoundly modify its development relative to the simple attached expansion characteristic of conventional axisymmetric nozzles. Understanding and controlling the boundary layer behavior is essential for achieving the mixing enhancement objectives of the present invention without incurring the severe performance penalties associated with flow separation.

Upon entering the corrugated section, the boundary layer immediately begins responding to the emerging circumferential pressure gradient. In regions approaching lobe peaks, the boundary layer experiences favorable pressure gradient as flow accelerates toward the locally converging geometry. This favorable gradient thins the boundary layer and increases the near-wall velocity, enhancing resistance to separation. In regions approaching lobe troughs, the boundary layer experiences adverse pressure gradient as flow decelerates in response to the locally diverging geometry. This adverse gradient thickens the boundary layer and reduces near-wall velocity, diminishing separation resistance.

The magnitude of the circumferential pressure gradient increases with axial position through the corrugated section as lobe amplitude grows. At the corrugation initiation station where amplitude equals zero, no circumferential gradient exists and the boundary layer continues its axisymmetric development. At the exit plane where amplitude reaches maximum value, the circumferential pressure gradient reaches approximately zero point three bar per radian, creating substantial local variations in boundary layer properties.

The boundary layer momentum thickness at the exit plane varies circumferentially from approximately zero point four five millimeters at lobe peaks to approximately zero point seven five millimeters at lobe troughs. This factor-of-one-point-seven variation in momentum thickness around the circumference reflects the integrated history of favorable and adverse pressure gradients experienced by different circumferential portions of the boundary layer as they traverse the corrugated section.

The shape factor provides a sensitive indicator of separation susceptibility, with values exceeding approximately three point zero indicating impending separation for turbulent boundary layers. At lobe peak locations, the shape factor decreases through the corrugated section from its initial value of two point six, reaching approximately two point three at the exit plane. This decrease reflects the stabilizing influence of the favorable pressure gradient and streamline convergence at peak locations. At lobe trough locations, the shape factor increases through the corrugated section, potentially approaching critical values if the divergence angle exceeds acceptable limits.

The critical constraint on corrugation geometry arises from the requirement to maintain shape factor below separation threshold throughout the corrugated section. This constraint translates into limits on the local wall divergence angle at trough locations, which in turn constrains the rate of amplitude growth. For the representative implementation, the maximum wall divergence angle at trough locations equals twelve degrees, occurring near the exit plane where amplitude growth rate is greatest. This angle provides adequate margin below the separation angle of approximately fourteen degrees established by experimental investigation of turbulent boundary layers under comparable conditions.

The sixty-percent corrugation extent directly influences the achievable divergence angles by determining the axial distance available for amplitude growth. Reducing the corrugation extent below sixty percent while maintaining the same exit plane amplitude would require more aggressive amplitude growth rates and correspondingly steeper divergence angles, potentially exceeding the separation threshold and precipitating the boundary layer detachment that would severely degrade nozzle performance.

The interaction between shock waves and the boundary layer creates additional separation vulnerability at specific locations within the corrugated section. Where oblique shock waves impinge upon the boundary layer at the wall surface, the sudden pressure rise across the shock can trigger separation even when the underlying geometric pressure gradient would maintain attached flow. The most critical shock-boundary layer interaction locations occur at the wall opposite from the lobe peaks, where the peak-generated oblique shocks reach the wall after crossing the central flow region.

For the eight-lobe configuration, shock impingement on the wall at trough locations begins at approximately twenty percent of the corrugation extent downstream of the initiation station, where the developing shock waves first reach sufficient strength and propagation distance to interact with the opposite wall. The impingement pressure rise at this initial contact equals approximately one point one five times the upstream pressure, insufficient to cause separation of the healthy boundary layer prevailing at this location.

As the flow progresses downstream and shock strength increases, the impingement pressure ratio increases and the separation margin decreases. At the exit plane, the shock impingement pressure ratio at trough locations reaches approximately one point four, which would cause separation of a boundary layer in the incipient separation condition but is resisted by the attached boundary layer present due to the favorable axial pressure gradient of supersonic expansion.

The three-dimensional character of the boundary layer within the corrugated section creates cross-flow velocity components that transport momentum circumferentially around the lobe contours. This cross-flow, driven by the circumferential pressure gradient, tends to convect high-momentum fluid from peak regions toward trough regions, partially compensating for the adverse pressure gradient effects at the troughs. The cross-flow magnitude reaches approximately fifteen percent of the primary axial velocity at positions near the wall in the mid-circumference regions between peaks and troughs.

The beneficial cross-flow transport is most effective when the corrugated section provides sufficient axial distance for the three-dimensional boundary layer equilibration to develop. For corrugation extents significantly less than sixty percent, the cross-flow has insufficient development length to achieve substantial momentum transport before the flow exits the nozzle. For corrugation extents significantly greater than sixty percent, the extended exposure to three-dimensional pressure gradients can cause the cross-flow to become excessive, creating locally separated regions at the downstream portions of trough side walls where the cross-flow must decelerate.

The thermal environment within the corrugated section exhibits three-dimensional variations that reflect the complex flow physics occurring in this region. The non-uniform heating distribution creates design challenges for the thermal management system while simultaneously providing opportunities for optimization through exploitation of the natural cooling afforded in lower-heating regions.

The convective heat transfer coefficient at the nozzle wall depends upon the local flow velocity, density, and temperature, as well as the boundary layer thickness and turbulence intensity. At lobe peak locations, the locally favorable pressure gradient creates thinner boundary layers with steeper near-wall velocity gradients, resulting in higher heat transfer coefficients. At lobe trough locations, the thicker boundary layers produce lower heat transfer coefficients despite the nominally similar freestream conditions.

The heat transfer coefficient variation around the circumference at the exit plane spans from approximately four thousand watts per square meter per Kelvin at lobe peaks to approximately two thousand five hundred watts per square meter per Kelvin at lobe troughs, representing a factor-of-one-point-six variation. This circumferential variation must be accommodated by the thermal management system to maintain acceptable wall temperatures at peak locations without overcooling at trough locations.

Shock wave impingement on the wall surface creates localized regions of intensified heating where the boundary layer compression through the shock increases local temperature and density while reducing boundary layer thickness. The impingement heating augmentation reaches maximum values at the intersection between the shock foot and the wall surface, with gradual recovery to undisturbed levels over a distance of approximately five to ten boundary layer thicknesses downstream of the impingement location.

For the representative implementation, shock impingement heating augmentation at trough locations increases local heat transfer coefficient by factors ranging from one point three near the corrugation initiation station to one point eight near the exit plane, reflecting the strengthening shock waves as lobe amplitude grows. The augmented heat transfer coefficients at shock impingement locations reach approximately four thousand five hundred watts per square meter per Kelvin near the exit, comparable to or exceeding the steady-state peak values.

The driving temperature difference for convective heat transfer equals the difference between the adiabatic wall temperature and the actual wall temperature maintained by the cooling system. The adiabatic wall temperature represents the equilibrium temperature that an insulated wall would achieve under the prevailing flow conditions, accounting for the recovery of kinetic energy in the boundary layer. For the high-speed flow within the corrugated section, the adiabatic wall temperature exceeds the static temperature by several hundred Kelvin due to kinetic energy recovery.

At the exit plane, the static temperature of the expanding combustion gases has decreased to approximately one thousand six hundred Kelvin, while the adiabatic wall temperature remains at approximately two thousand one hundred Kelvin due to the recovery factor of approximately zero point nine characteristic of turbulent boundary layers. The cooling system maintains wall temperature at approximately six hundred fifty Kelvin at peak locations where heating is most intense and at approximately five hundred fifty Kelvin at trough locations where heating is reduced.

The heat flux at the wall surface equals the product of heat transfer coefficient and driving temperature difference. At lobe peak locations near the exit plane, the heat flux reaches approximately six megawatts per square meter, requiring vigorous regenerative cooling to maintain acceptable wall temperatures. At lobe trough locations, the reduced heat transfer coefficient and lower wall temperature combine to produce heat flux of approximately four megawatts per square meter, still substantial but more readily managed.

The total heat rejection rate from the corrugated section of the representative implementation equals approximately two point five megawatts, distributed non-uniformly over the wall surface with concentration at peak locations and shock impingement regions. This heat rejection rate must be accommodated by the regenerative cooling system through appropriate coolant flow rate, flow velocity, and channel geometry distributions that match the local cooling capability to the local heating intensity.

The regenerative cooling system for the corrugated section employs a network of channels that route propellant around the three-dimensional lobe contours while providing enhanced cooling effectiveness in regions of concentrated thermal loading. The channel geometry and coolant flow distribution are specifically tailored to the unique thermal environment of the corrugated lobe configuration.

The cooling channels originate at a manifold located at the nozzle exit plane, receiving liquid hydrocarbon propellant (RP-1 kerosene for the representative implementation) at temperature of approximately two hundred ninety Kelvin and pressure of approximately eighty-five bar. The propellant flows upstream through the channels in counterflow arrangement relative to the combustion gas flow direction, absorbing heat from the wall and achieving progressive temperature increase before exiting at a manifold located upstream of the corrugation initiation station.

The channel cross-section within the corrugated section varies both axially and circumferentially to achieve the required distribution of coolant velocity and heat transfer capability. At lobe peak locations, channel width equals one point eight millimeters and channel depth equals two point two millimeters, providing cross-sectional area of approximately four square millimeters. The reduced channel area at peak locations accelerates the coolant flow, achieving local coolant velocity of approximately twenty-five meters per second compared to the average velocity of approximately eighteen meters per second through the overall cooling circuit.

At lobe trough locations, channel width increases to two point four millimeters and channel depth increases to two point eight millimeters, providing cross-sectional area of approximately six point seven square millimeters. This increased area permits lower coolant velocity of approximately fifteen meters per second at trough locations, reducing pumping power requirements in regions where the heating intensity does not require maximum cooling effectiveness.

The channel walls separating adjacent cooling passages provide structural continuity between the inner hot-gas-side wall and the outer closeout wall. The channel wall width equals one point zero millimeter throughout the corrugated section, providing adequate structural capability while minimizing the fin efficiency losses associated with heat conduction through extended channel walls. The ratio of channel width to channel wall width plus channel width, termed the channel fraction, varies from approximately zero point six four at peak locations to approximately zero point seven one at trough locations.

The heat transfer from the hot wall surface to the flowing coolant occurs through a series of thermal resistances comprising conduction through the wall thickness, convection to the coolant in the channel base, and additional convection to the coolant through the channel sidewalls acting as extended fins. The overall thermal resistance from hot gas to coolant determines the wall temperature achieved for given heat flux and coolant temperature conditions.

At lobe peak locations where heat flux is maximum, the thermal design provides wall-to-coolant temperature difference of approximately three hundred fifty Kelvin at the exit plane, where coolant temperature equals approximately three hundred Kelvin after modest heating through the downstream portions of the cooling circuit. This temperature difference, combined with the local heat transfer coefficient and wall thickness, maintains hot-side wall temperature at approximately six hundred fifty Kelvin as previously specified.

At lobe trough locations, the reduced heat flux and increased cooling channel area combine to produce wall-to-coolant temperature difference of approximately two hundred fifty Kelvin, maintaining hot-side wall temperature at approximately five hundred fifty Kelvin. This lower temperature at trough locations provides additional margin for the somewhat thicker walls required in these regions for structural reasons.

The coolant pressure drop through the corrugated section totals approximately three point five bar for the representative implementation, distributed between frictional losses along the channel length and losses through the entrance and exit manifolds. The non-uniform velocity distribution creates corresponding non-uniformity in pressure drop, with the high-velocity peak channels experiencing somewhat greater pressure drop than the low-velocity trough channels. Manifold design accommodates this non-uniformity through appropriate sizing of the distribution passages.

The coolant temperature rise through the corrugated section equals approximately one hundred twenty Kelvin, increasing coolant temperature from approximately three hundred Kelvin at the exit plane manifold to approximately four hundred twenty Kelvin at the upstream manifold. This temperature rise corresponds to absorption of approximately two point five megawatts at coolant flow rate of approximately eleven kilograms per second and specific heat of approximately two thousand joules per kilogram per Kelvin. The heated coolant proceeds upstream through the smooth divergent section and throat region cooling circuits before injection into the combustion chamber.

The shock impingement regions present particular thermal management challenges due to the localized heating augmentation that exceeds the capabilities of the standard regenerative cooling provisions. The present invention incorporates enhanced cooling features at these locations to prevent excessive wall temperatures while avoiding the complexity and mass penalty of uniformly upgraded cooling throughout the corrugated section.

The shock impingement locations are predictable from the lobe geometry and flow conditions, enabling targeted placement of enhanced cooling features. For the eight-lobe configuration, shock impingement occurs at trough locations along eight axial lines positioned at the circumferential centers of the eight troughs. The impingement begins at approximately twenty percent of the corrugation extent downstream of the initiation station and extends to the exit plane, with impingement intensity increasing in the downstream direction as shock strength grows.

The enhanced cooling at shock impingement locations employs turbulence promoters within the cooling channels that increase local heat transfer coefficient through disruption of the thermal boundary layer. Transverse ribs machined into the channel base at two millimeter intervals trip the coolant boundary layer, creating local flow separation and reattachment patterns that enhance mixing between the near-wall coolant and the cooler core flow.

The turbulence promoter rib height equals zero point three millimeters, approximately ten percent of the channel depth, providing effective boundary layer disruption without excessive flow blockage that would cause unacceptable pressure drop increase. The ribs extend across the full channel width with square cross-section profile. The resulting heat transfer augmentation reaches a factor of approximately two point two relative to smooth channel performance, enabling accommodation of the shock-induced heating augmentation without requiring increased coolant flow rate or channel geometry modification.

The turbulence promoters are positioned along an axial extent of approximately one hundred millimeters at each shock impingement location, centered on the predicted impingement line and spanning sufficient circumferential extent to accommodate the finite width of the shock-affected zone. The promoters terminate upstream and downstream of the shock impingement region to avoid unnecessary pressure drop penalty where heating augmentation is not required.

The aggregate pressure drop contribution of the turbulence promoters equals approximately zero point eight bar, increasing total cooling circuit pressure drop to approximately four point three bar. This additional pressure drop is accommodated within the propellant feed system design through appropriate turbopump discharge pressure specification.

Alternative embodiments of the enhanced cooling for shock impingement regions employ impingement cooling, wherein coolant jets directed perpendicular to the wall surface create localized high-intensity cooling at the impingement points. This approach provides potentially superior heat transfer performance but requires more complex channel geometry including dedicated supply passages and spent coolant removal provisions. The impingement cooling approach is particularly advantageous for applications requiring exceptionally high wall heat flux tolerance.

The structural design of the corrugated lobe section addresses the unique loading patterns arising from three-dimensional supersonic flow through the non-axisymmetric geometry. The loading sources include internal pressure from the combustion gases, thermal stress from temperature gradients through the wall thickness and around the circumference, and inertial loads from vehicle acceleration.

The internal pressure loading creates membrane stresses in the wall that resist the tendency of the pressure to expand the structure radially. For axisymmetric cylinders, the hoop stress equals the product of pressure, radius, and wall thickness inverse. For the non-circular cross-sections of the corrugated geometry, the local hoop stress varies circumferentially in inverse proportion to the local radius of curvature.

At lobe peak locations, the local radius of curvature in the circumferential direction exceeds the mean radius due to the outward bulge of the peak geometry. For the representative implementation with eighteen millimeter amplitude at the exit plane and mean radius of one hundred sixty-two millimeters, the radius of curvature at peak centers equals approximately two hundred ten millimeters, creating local hoop stress reduction of approximately twenty-three percent relative to an axisymmetric design of equivalent mean radius and wall thickness.

At lobe trough locations, the local radius of curvature falls below the mean radius due to the inward indentation of the trough geometry. The radius of curvature at trough centers equals approximately one hundred twenty-five millimeters, creating local hoop stress increase of approximately thirty percent relative to axisymmetric conditions. This stress concentration at trough locations necessitates the increased wall thickness specified for these regions to maintain consistent stress levels around the circumference.

The internal pressure within the corrugated section varies axially from approximately five bar near the corrugation initiation station to approximately one point two bar at the exit plane, following the supersonic expansion process. The circumferential pressure variation due to shock and expansion effects adds local variations of approximately plus or minus zero point three bar to the mean axial value. The structural analysis considers the worst-case combination of mean pressure and circumferential variation at each axial station.

The maximum hoop stress at the exit plane, where pressure is lowest but wall thickness is also reduced, equals approximately seventy megapascals for the representative implementation. This stress level occurs at trough locations where the increased thickness partially compensates for the curvature-induced stress concentration. At peak locations, the reduced stress due to favorable curvature combines with the thinner wall to produce comparable stress levels.

Thermal stresses arise from the temperature gradients that inevitably accompany the heat transfer process from hot combustion gases to regenerative coolant. The temperature gradient through the wall thickness creates differential expansion between the hot inner surface and the cooler outer surface, generating bending stresses that add to the membrane stresses from pressure loading.

For the representative wall thickness of three point five millimeters and temperature difference of approximately two hundred Kelvin through the thickness at peak locations, the thermal bending stress equals approximately sixty megapascals. This thermal stress adds algebraically to the pressure-induced membrane stress, with the sign depending upon whether the thermal gradient creates tension or compression at the surface of interest.

At the hot inner surface, the thermal stress component is compressive as the constrained hot material attempts to expand against the cooler outer layers. At the cooler outer surface, the thermal stress component is tensile. The total stress at the inner surface equals the membrane tension minus the thermal compression, while the total stress at the outer surface equals the membrane tension plus the thermal tension. The outer surface therefore experiences the maximum total tensile stress.

The circumferential temperature variation between peak and trough locations creates additional thermal stress through the differential expansion of adjacent regions at different temperatures. The temperature difference of approximately one hundred Kelvin between peak and trough at the exit plane creates circumferential thermal stress of approximately forty megapascals, adding to the other stress components at the transition regions between peaks and troughs.

The material selection for the corrugated section balances the competing requirements of high-temperature strength, thermal conductivity, oxidation resistance, and compatibility with the manufacturing processes. For the representative implementation operating with hydrocarbon fuel cooling and moderate exit plane wall temperatures, the selected material is Inconel 718 nickel-base superalloy. This material provides yield strength exceeding three hundred megapascals at operating temperature, thermal conductivity of approximately eighteen watts per meter per Kelvin enabling effective heat transfer to the coolant, and excellent resistance to oxidation and hot corrosion.

Alternative materials for applications with different requirements include copper alloys for enhanced thermal conductivity when cooling margin is critical, refractory metals such as niobium or molybdenum alloys for applications requiring higher wall temperature tolerance, and ceramic matrix composites for applications pursuing maximum temperature capability with reduced cooling requirements.

The fatigue life analysis addresses the cyclic loading experienced during repeated engine firings over the intended operational lifetime. The stress cycles from zero (engine off) to maximum operating stress (full thrust) with each firing, with additional smaller cycles from thrust transients during startup and shutdown. For the representative implementation designed for one hundred firing cycles with factor of safety of four, the allowable alternating stress at mean stress of one hundred megapascals equals approximately one hundred twenty megapascals based on the Inconel 718 fatigue properties at operating temperature.

The actual alternating stress equals one-half the stress range from zero to maximum, or approximately sixty-five megapascals for the maximum total stress of one hundred thirty megapascals at the outer surface. The comparison of actual alternating stress to allowable alternating stress confirms adequate fatigue life margin for the intended application.

The complex three-dimensional geometry of the corrugated lobe section is manufactured using selective laser melting additive manufacturing technology, wherein successive thin layers of metal powder are selectively melted by a focused laser beam to build up the complete component geometry. This manufacturing approach provides inherent capability for complex internal features including the regenerative cooling channels that would require extensive machining operations if produced by conventional methods.

The manufacturing begins with generation of the digital solid model representing the complete corrugated section geometry including the three-dimensional outer surface, the precisely-defined inner flow surface, and the internal cooling channel network. The solid model is created using computer-aided design software with the geometric parameters specified elsewhere in this description encoded through parametric feature definitions. The model is subsequently processed through specialized build preparation software that slices the geometry into thin horizontal layers (typically thirty to fifty micrometers thickness) and generates the laser scan paths for each layer.

The build process employs Inconel 718 powder with particle size distribution ranging from fifteen to forty-five micrometers. The powder is produced by gas atomization of prealloyed ingot material, creating spherical particles with controlled composition and minimal satellite particle attachment. The powder is characterized prior to use through particle size analysis, chemical composition verification, and flow behavior testing to confirm suitability for the build process.

The selective laser melting machine employed for component production incorporates a build chamber with dimensions sufficient to accommodate the corrugated section component (build volume of at least five hundred millimeters diameter by four hundred fifty millimeters height for the representative implementation). The build chamber is maintained under high-purity argon atmosphere with oxygen content below fifty parts per million to prevent oxidation of the molten metal during processing.

The laser system provides beam power of four hundred to one thousand watts with spot size of approximately eighty micrometers at the powder bed surface. The laser wavelength of one thousand seventy nanometers couples efficiently with the Inconel 718 powder, achieving greater than forty percent absorption of incident energy. The scanning system enables beam positioning accuracy of ten micrometers and maximum scanning velocity of seven meters per second, though typical processing velocity ranges from eight hundred to one thousand two hundred millimeters per second for the optimal melt pool characteristics.

The build process proceeds layer by layer, with each cycle comprising powder deposition, laser scanning, and platform lowering. The recoater blade or roller spreads a uniform layer of powder across the build surface at the specified layer thickness. The laser then scans the regions corresponding to solid material in the current layer slice, melting the powder and fusing it to the previously solidified layer below. After scanning is complete, the build platform lowers by the layer thickness and the next powder layer is deposited.

The scanning strategy for each layer employs a combination of contour scans and hatching scans optimized for the specific geometric features present. Contour scans trace the outer and inner boundaries of the solid regions, establishing the surface geometry with minimum deviation from the intended contour. Hatching scans fill the interior regions with overlapping melt tracks oriented at angles that change between layers to avoid directional texture development. The hatch spacing of approximately eighty micrometers provides sufficient overlap between adjacent melt tracks to achieve fully dense material.

The internal cooling channels are created through absence of laser scanning in the channel regions, leaving unfused powder that is subsequently removed. The channel geometry must accommodate the powder removal requirement, avoiding enclosed cavities and providing adequate access for powder extraction. For the representative implementation, the channels connect to powder removal ports at both manifold locations, enabling extraction of unfused powder through vibration-assisted drainage and vacuum suction.

The total build time for the corrugated section component equals approximately sixty-five hours for the representative implementation, including all layers from the exit plane manifold through the corrugation initiation station junction. The build proceeds with the component axis oriented vertically and the exit plane facing downward, minimizing the overhang angles for the lobe peak surfaces and reducing the support structure requirements.

Support structures are required for surfaces oriented at angles less than approximately forty-five degrees from horizontal to prevent distortion and collapse of the overhanging features. For the corrugated section geometry, support structures are generated beneath the lobe peak surfaces where the wall angle approaches horizontal at the peaks. The support structures are designed for subsequent removal through mechanical means, with access provided through the open exit plane and the channel manifold locations.

Following build completion, the component remains attached to the build plate through the support structure foundation. Initial powder removal is performed while the component remains attached, using vibration and suction to extract the unfused powder from the internal channels and from the open regions surrounding the component. Powder removal verification employs computed tomographic scanning or endoscopic inspection to confirm complete extraction from all internal passages.

The component is then separated from the build plate through wire electrical discharge machining, cutting through the support structure foundation to release the free-standing component. The residual support structure material attached to the component is removed through a combination of manual breaking, grinding, and localized machining, with care taken to avoid damage to the functional surfaces.

Heat treatment is performed to relieve residual stresses from the build process and to achieve the specified mechanical properties. The standard heat treatment for Inconel 718 comprises solution annealing at nine hundred fifty to one thousand degrees Celsius followed by two-step aging at seven hundred twenty degrees Celsius and six hundred twenty degrees Celsius. This heat treatment precipitates the gamma-prime and gamma-double-prime strengthening phases that provide the high-temperature strength required for the propulsion application.

Post-build machining operations are performed on critical surfaces where the as-built surface finish or dimensional tolerance is insufficient. The inner flow surface of the corrugated section requires surface finish better than six point three micrometers Ra to minimize boundary layer transition effects and flow losses. The as-built surface finish of fifteen to twenty micrometers Ra is improved through abrasive flow machining, wherein abrasive-laden viscoelastic media is forced through the internal passage to remove surface irregularities.

The manifold interface surfaces require machining to achieve the flatness and surface finish necessary for sealing and alignment with adjacent components. These surfaces are machined using multi-axis computer numerical control milling with cutting parameters appropriate for the hardened Inconel 718 material. The manifold bolt hole patterns are produced by drilling and reaming to achieve the positional accuracy and hole quality required for the fastener installation.

The quality assurance program for the corrugated section component encompasses incoming material verification, in-process monitoring, and post-build inspection to ensure compliance with all dimensional, structural, and functional requirements. The program is designed to detect both systematic process variations and random defects that could compromise component performance or reliability.

Incoming powder material is inspected for chemical composition, particle size distribution, particle morphology, and flow characteristics prior to acceptance for production use. Chemical composition is verified through inductively-coupled plasma optical emission spectrometry for major alloying elements and combustion analysis for carbon and sulfur content. The composition must fall within the ranges specified by the Inconel 718 material specification, with particular attention to elements affecting weldability such as boron, sulfur, and phosphorus.

Particle size distribution is measured through laser diffraction analysis, confirming that the D10, D50, and D90 values (representing the particle sizes at which ten, fifty, and ninety percent of the distribution falls below) meet the specified requirements. Excessively fine particles can cause powder bed density problems and spattering during laser processing, while excessively coarse particles create surface roughness issues and may not fully melt during the scanning process.

Powder morphology is examined through scanning electron microscopy, verifying the spherical shape and absence of satellite particles that would compromise powder flow and packing characteristics. Powder flow behavior is quantified through Hall flowmeter testing per ASTM B213, with flow rate requirements ensuring consistent powder bed deposition during the build process.

In-process monitoring during the additive build employs multiple sensing systems to detect anomalies that could indicate developing defects or process drift. Melt pool monitoring using photodiode and high-speed camera systems observes the emission intensity and geometry of the melt pool during laser scanning. Deviations from the expected melt pool characteristics, such as spattering, keyholing, or insufficient melting, trigger alerts for operator evaluation and potential process parameter adjustment.

Layer imaging after each powder deposition captures the powder bed surface condition before laser scanning. Image analysis algorithms detect recoating defects, powder clumps, and other anomalies that could cause layer-to-layer defects in the solidified material. Layer imaging after laser scanning detects surface irregularities, incomplete fusion, and geometric deviations from the expected layer boundaries.

Build plate temperature monitoring ensures consistent thermal conditions throughout the extended build time. The build plate temperature is maintained at approximately eighty degrees Celsius to reduce residual stress development while avoiding powder cake formation that would complicate powder removal. Temperature variations exceeding specified limits trigger process interruption for investigation.

Post-build inspection begins with comprehensive dimensional verification using computed tomographic scanning. The CT scan acquires complete three-dimensional density data throughout the component volume, enabling reconstruction of both internal and external geometries. Dimensional analysis compares the reconstructed geometry with the design intent, quantifying deviations in wall thickness, channel dimensions, lobe profile, and overall dimensional characteristics.

The CT scan data simultaneously enables detection of internal defects including porosity, lack of fusion, cracking, and entrapped unfused powder. The acceptance criteria specify maximum defect sizes and defect population densities that are demonstrated to be compatible with the structural and fatigue requirements. Defects exceeding the acceptance criteria require disposition through engineering evaluation and potential component rejection.

Surface inspection of the inner flow surface employs borescopic examination following surface finish improvement through abrasive flow machining. The borescope provides magnified visual inspection of the surface condition, detecting residual roughness, machining artifacts, or surface damage that could affect aerodynamic performance. Surface roughness is quantified through profilometer measurements at accessible locations, with correlation to full-surface roughness distribution based on process characterization data.

Leak testing verifies the integrity of the cooling channel walls, confirming the absence of through-wall defects that would permit combustion gas ingestion into the cooling circuit or coolant leakage into the flow path. The test procedure applies helium gas to the cooling circuit at pressure exceeding the maximum operating coolant pressure while the flow path interior is maintained under vacuum. Helium mass spectrometer detection at the flow path provides sensitivity to leakage rates as low as ten to the negative ninth standard cubic centimeters per second, ensuring detection of even minute through-wall paths.

Proof pressure testing demonstrates structural capability by applying hydrostatic pressure to the cooling circuit at one hundred fifty percent of maximum operating pressure. The test pressure is held for duration of at least five minutes while monitoring for leakage, permanent deformation, or rupture. Successful completion of proof testing without any of these failure modes demonstrates adequate structural capability for the intended application.

Material property verification employs test coupons built simultaneously with the production component using identical processing parameters. The test coupons undergo tensile testing, hardness testing, and metallographic examination to confirm that the material properties achieved in the production build meet all specification requirements. The test coupon location on the build plate and test coupon orientation relative to the build direction are selected to provide representative sampling of the property variations occurring within the production component.

The corrugated section component is integrated with the other propulsion system elements through mechanical and fluidic interfaces that accommodate the thermal expansion, pressure loading, and assembly tolerance stack-up occurring in the complete system. The integration process ensures proper alignment, sealed interfaces, and structural continuity throughout the nozzle assembly.

The upstream interface connects the corrugated section to the smooth divergent section through a bolted flange joint with metal seal. The flange faces are machined to provide flatness within twenty-five micrometers to ensure uniform seal loading around the circumference. The metal seal, fabricated from silver-plated Inconel 718 material, deforms plastically under bolt preload to conform to minor surface irregularities and provide reliable sealing against both coolant leakage and combustion gas intrusion.

The bolt pattern comprises twenty-four M8 bolts on a bolt circle diameter of two hundred forty millimeters, providing sufficient clamping force to maintain seal compression under all operating and transient conditions. The bolt preload of twenty-five kilonewtons per bolt creates interface compression of approximately two hundred megapascals at the seal location, exceeding the yield strength of the silver plating and achieving the required plastic deformation for effective sealing.

The coolant flow interfaces at the upstream and downstream manifolds connect to the propulsion system coolant circuit through welded tube joints. The manifold ports accept tube stubs of fifteen millimeters outer diameter and one point five millimeters wall thickness, fabricated from the same Inconel 718 material as the corrugated section component. The tubes are joined by orbital gas tungsten arc welding with automated travel speed and wire feed to achieve consistent weld quality around the circumference.

Weld inspection employs radiographic examination per aerospace welding quality standards, verifying complete fusion, absence of porosity or cracking, and proper weld profile geometry. Liquid penetrant inspection of the external weld surface detects surface-breaking defects that might not be visible in the radiographic examination.

The assembled nozzle undergoes system-level proof testing that verifies the integrity of all joints and interfaces under combined pressure and thermal loading. The proof test applies coolant pressure and flow while the inner flow surface is pressurized with inert gas to simulate the combustion gas loading. Temperature simulation is achieved through electrical resistance heating of the inner surface, creating wall temperature gradients representative of operating conditions.

Functional testing of the cooling circuit verifies proper flow distribution through measurement of pressure drop and outlet temperature distribution. Flow visualization using tracer injection and observation at manifold outlets confirms that all cooling channels receive proper flow without blockage or excessive bypassing. Thermal response testing measures the transient temperature behavior during simulated startup and shutdown conditions, verifying adequate cooling capacity under all operating modes.

Hot-fire testing of the complete propulsion system provides the final verification of corrugated section performance under actual operating conditions. The initial hot-fire tests employ short-duration firings (two to five seconds) at reduced chamber pressure to verify basic functionality and stability before progressing to full-duration firings at full thrust. Instrumentation including thermocouples at multiple wall locations, pressure transducers in the cooling circuit, and high-speed photography of the exhaust plume provides comprehensive data for performance verification and model validation.

The characteristic shock patterns predicted for the corrugated lobe geometry are verified through schlieren photography or shadowgraph imaging of the exhaust plume during hot-fire testing. The double-diamond to single-diamond shock coalescence behavior is observable in the plume images, confirming that the design intent has been achieved and the analytical predictions are validated by the actual hardware performance.

Thrust measurement during hot-fire testing verifies that the corrugated lobe configuration achieves the expected performance relative to conventional nozzle designs. The mixing enhancement and shock management benefits must be realized without unacceptable sacrifice in propulsive efficiency, as confirmed by comparison of measured specific impulse with predictions and with reference data from equivalent conventional nozzle tests.

The operational use of the propulsion system incorporates the corrugated lobe section as an integrated element of the complete launch vehicle propulsion architecture. The operational procedures address all phases of use from pre-flight checkout through launch, flight, and post-flight assessment.

Pre-flight checkout of the corrugated section component focuses on verification of cooling circuit integrity and cleanliness. Leak testing using helium trace gas confirms the continued integrity of all coolant passage walls and interface seals following any vehicle handling or transportation. Borescopic examination of the inner flow surface verifies the absence of foreign object debris that could affect flow quality or create downstream damage.

Coolant circuit conditioning removes any moisture or contamination that may have accumulated during storage or handling. The conditioning procedure flows dry nitrogen through the cooling passages at elevated velocity, entraining and removing particulate matter while the low dew point of the nitrogen promotes evaporation of any moisture present. Circuit cleanliness is verified through particle count measurement on the conditioning effluent.

Launch countdown procedures include thermal conditioning of the nozzle assembly to achieve uniform temperature distribution prior to ignition. For cryogenic propellant systems, the thermal conditioning addresses the significant temperature gradients that develop between cryogenic-wetted components and ambient-temperature elements. The corrugated section, located downstream of the main cryogenic contact regions, typically equilibrates with local ambient conditions but may require attention to prevent condensation if humid launch site conditions exist.

Engine start sequence initiates coolant flow prior to ignition to establish the thermal protection before combustion gases contact the wall surfaces. The coolant flow reaches full design flow rate within approximately fifty milliseconds of the start command, with ignition following approximately two hundred milliseconds later. This sequencing ensures that the thermal protection is fully established before the most thermally demanding conditions occur during the initial ignition transient.

During powered flight, the corrugated section operates within the thermal and structural design envelope established through the analysis and testing described previously. The wall temperatures remain within the demonstrated capability of the materials and cooling system, with transient excursions during thrust transients remaining within acceptable limits. The shock patterns evolve as external ambient pressure decreases during ascent, but the fundamental double-diamond to single-diamond coalescence behavior persists throughout the atmospheric flight phase.

At stage separation for multi-stage vehicles, the lower-stage engines shut down through propellant valve closure. The cooling flow continues briefly after combustion termination to remove residual heat from the nozzle walls before coolant flow is also terminated. The thermal mass of the nozzle structure absorbs the residual heat without excessive temperature rise during the post-shutdown thermal soak.

Upper-stage ignition occurs after stage separation, with the upper-stage corrugated section designed for the vacuum operating environment. The absence of ambient pressure eliminates the external flow interactions that influence lower-stage performance, and the corrugation parameters are optimized accordingly as described elsewhere in this disclosure.

Post-flight assessment for reusable propulsion systems includes inspection of the corrugated section for any degradation or damage accumulated during the mission. Visual examination of accessible surfaces detects any evidence of erosion, thermal damage, or foreign object impact. Leak testing verifies continued cooling circuit integrity. Dimensional verification confirms that no permanent deformation has occurred under the flight loading conditions.

Refurbishment procedures address any wear or damage identified during post-flight assessment. Minor surface irregularities on the inner flow surface are corrected through localized polishing or abrasive flow processing. More significant damage may require component replacement, with the modular assembly design facilitating removal and replacement of individual elements without complete propulsion system disassembly.

The variable geometry embodiment of the present invention incorporates sliding sleeve mechanisms that enable in-flight adjustment of the effective corrugation extent, optimizing nozzle characteristics for the varying conditions encountered during atmospheric transit. This section provides detailed description of the variable geometry mechanisms, actuation systems, and operational procedures.

The sliding sleeve assembly comprises eight sleeve segments, one positioned at each lobe trough location. Each sleeve segment spans approximately thirty degrees of azimuthal arc, covering the trough region between adjacent lobe peaks. The sleeve segments are independently movable in the axial direction, sliding along guide rails that extend parallel to the nozzle axis at the trough locations.

Each sleeve segment has length of two hundred fifty millimeters when fully extended, sufficient to cover the complete corrugated section from the corrugation initiation station to the exit plane. The sleeve segments are fabricated from Inconel 718 sheet material of two millimeters thickness, formed to match the trough contour of the corrugated section when in the retracted position. When extended, the sleeves present a smooth inner surface that covers the trough corrugation while leaving the peak regions exposed.

The guide rail system comprises pairs of rails positioned at each lobe sidewall, providing four rails per sleeve segment for stable guidance and prevention of circumferential rotation. The rails are fabricated from stellite cobalt alloy, providing the wear resistance necessary for repeated sleeve translation while maintaining low friction coefficient against the Inconel sleeve material. The rail profile includes labyrinth seals that minimize hot gas intrusion into the guide mechanism during operation.

The actuation system employs pneumatic cylinders operating on high-pressure helium supplied from the vehicle pneumatic system. Each sleeve segment is driven by two actuator cylinders positioned at the upstream and downstream ends of the sleeve travel range. The dual-cylinder arrangement provides redundancy against single actuator failure and enables differential force application for sleeve alignment during translation.

The pneumatic cylinders have bore diameter of twenty millimeters and stroke of two hundred sixty millimeters, providing translation range exceeding the sleeve length. The maximum operating pressure of two hundred bar generates actuator force of approximately six thousand three hundred newtons per cylinder, substantially exceeding the forces required for sleeve translation under operating conditions.

Position sensing employs linear variable differential transformers mounted parallel to each actuator cylinder. The transformer core is attached to the sleeve segment and moves within the transformer coils as the sleeve translates. The transformer output provides absolute position measurement with resolution of zero point one millimeters and accuracy of zero point five millimeters across the full travel range.

The control system receives commands from the vehicle flight computer specifying the desired sleeve position as a function of flight condition. The controller compares the commanded position with the measured position from each sleeve segment and generates the appropriate valve commands to apply pneumatic pressure for sleeve translation. Proportional control enables continuous positioning at any point within the travel range, not merely at the fully extended or fully retracted extremes.

The thermal environment at the sleeve location challenges the mechanism design through the combination of high temperature and temperature variation during operation. When retracted, the sleeves are exposed to the full combustion gas temperature and convective heating. When extended, the sleeves receive reduced heating from the cooler flow in the trough regions while the unexposed peak regions continue to experience full heating.

Sleeve cooling is provided through conductive heat transfer to the guide rails, which are cooled by dedicated cooling passages integrated into the rail structure. The cooling passages connect to the main regenerative cooling circuit through small-diameter tubing that permits the relative motion between the fixed rails and the translating sleeves. The thermal time constant of the sleeve material limits the temperature excursions during transient operation, with the sleeve bulk temperature remaining within the material capability for all anticipated operational scenarios.

The sealing between sleeve segments and the adjacent structure prevents hot gas intrusion into the mechanism cavities while accommodating the relative motion of the translating sleeves. Ceramic fiber rope seals positioned at the sleeve leading and trailing edges compress against stationary seal surfaces as the sleeves translate, providing compliant sealing with minimal friction. The seals are replaceable elements designed for periodic replacement during vehicle refurbishment.

Operational procedures for the variable geometry system include pre-flight checkout of sleeve translation and position sensing, in-flight adjustment according to the programmed schedule, and post-flight verification of mechanism condition. The pre-flight checkout exercises each sleeve segment through the full travel range while verifying proper position feedback, appropriate actuation forces, and absence of binding or irregular motion.

The in-flight adjustment schedule specifies sleeve position as a function of altitude, with the sleeves typically in the fully retracted position at launch (exposing complete corrugation for maximum sea-level mixing benefit), progressively extending during atmospheric ascent as ambient pressure decreases, and reaching fully extended position (covering corrugation for vacuum-optimized smooth geometry) before stage separation or orbit insertion.

The transition rate is controlled to prevent rapid changes in nozzle characteristics that could create thrust transients affecting vehicle guidance. Typical translation rates of two millimeters per second result in complete transition from retracted to extended position over approximately two minutes, corresponding to the altitude

range from approximately fifteen to fifty kilometers where the transition provides greatest benefit.

Alternative embodiments employ mechanical linkages that provide sleeve translation as an automatic function of ambient pressure, eliminating the pneumatic actuation and electronic control complexity. These pressure-responsive mechanisms employ bellows or diaphragm elements exposed to ambient conditions, with mechanical linkages that convert the pressure-responsive motion to sleeve translation. The passive approach provides inherent reliability benefits while sacrificing the flexibility of programmable position control.

The corrugated lobe propulsion system of the present invention achieves performance advantages across multiple metrics relative to conventional smooth-wall nozzle designs. These advantages arise from the controlled manipulation of shock wave structures, enhanced mixing through vortical mechanisms, and improved adaptation to varying operating conditions.

The mixing enhancement provided by the streamwise vortical structures accelerates the equilibration between exhaust gases and ambient air or secondary propellant streams. For applications requiring rapid mixing such as air-augmented rocket propulsion, the mixing length required to achieve ninety percent thermal equilibration is reduced by approximately forty percent relative to equivalent round nozzles operating under comparable conditions.

The shock wave coalescence behavior reduces the acoustic emission intensity in the high-frequency bands where the small-scale shock oscillations would otherwise radiate substantial noise energy. The measured overall sound pressure level at reference distance shows reduction of approximately three decibels relative to conventional nozzle designs, with the reduction concentrated in frequency bands above two kilohertz where hearing damage risk is greatest.

The separation resistance afforded by the sixty-percent corrugation extent enables reliable operation at nozzle pressure ratios as low as sixty percent of the design value without the flow instabilities associated with separated nozzles. This expanded operating range reduces altitude compensation requirements for launch vehicle applications and increases operational flexibility for in-space propulsion applications with variable thrust requirements.

The thermal management advantages of the three-dimensional cooling channel geometry include reduced coolant pressure drop for equivalent cooling effectiveness and reduced peak wall temperatures through exploitation of secondary flow enhancement within curved channel passages. These advantages translate to reduced turbopump power requirements or increased thermal margin, either of which contributes to overall propulsion system improvement.

The structural efficiency of the corrugated geometry provides favorable ratio of wall material mass to contained pressure volume relative to equivalent circular cross-sections. The membrane stress reduction at lobe peaks permits local wall thickness reduction that partially compensates for the thickness increases required at lobe troughs, resulting in aggregate mass comparable to or below that of equivalent axisymmetric designs.

The manufacturing advantages of the additive manufacturing approach include the ability to produce the complete corrugated section with integrated cooling channels as a single monolithic component, eliminating the joints, welds, and assembly operations required for conventional multi-piece fabrication. This integration reduces manufacturing time, eliminates potential leak paths at joints, and ensures geometric consistency between internal cooling passages and external flow surfaces.

The present invention encompasses variations and alternative embodiments that adapt the fundamental corrugated lobe concept to different applications, propellant combinations, and performance requirements while maintaining the essential sixty-percent corrugation extent optimization.

Embodiments for hydrogen-oxygen propellant combinations accommodate the substantially higher specific impulse and correspondingly higher exit velocity and area ratio characteristic of these propellants. The throat diameter reduces for equivalent thrust magnitude, and the divergent section length increases to achieve the higher expansion ratios (typically twenty-five to forty for vacuum applications). The corrugation initiation station and corrugation extent scale proportionally with the increased divergent length, maintaining the sixty-percent ratio that provides optimal performance.

Embodiments for solid rocket motor applications adapt the corrugated lobe geometry for the ablative nozzle materials characteristic of these motors. The corrugation is machined or formed into the graphite, carbon-carbon composite, or silica phenolic ablative materials rather than into the metallic regeneratively-cooled structures of liquid propellant systems. The corrugation parameters are adjusted to account for the progressive surface erosion occurring during motor operation, ensuring that the corrugation remains effective as the surface contour evolves.

Embodiments for hybrid rocket motors combine features of liquid and solid motor approaches, with the oxidizer-wetted injector end employing metallic construction and the fuel-wetted chamber and nozzle regions employing ablative materials. The corrugated lobe geometry is implemented in the metallic nozzle extension where applicable, or in the ablative throat and divergent section following the solid motor approach.

Embodiments for electric propulsion systems adapt the corrugated lobe concept for the substantially different operating conditions of ion, Hall effect, or magnetoplasmadynamic thrusters. The corrugation functions to enhance mixing between the accelerated propellant stream and any neutral gas from imperfect ionization, improving thrust recovery from incompletely ionized propellant. The electromagnetic acceleration mechanisms of electric thrusters create fundamentally different shock and expansion patterns than chemical propulsion, and the corrugation parameters are correspondingly modified.

Embodiments for airbreathing propulsion, including ramjet and scramjet systems, employ the corrugated lobe geometry at the exhaust nozzle to enhance mixing between the combustion products and the bypass air stream. The different operating Mach number range and temperature conditions of airbreathing systems influence the corrugation parameter selection while maintaining the sixty-percent extent optimization.

Embodiments incorporating film cooling rather than regenerative cooling adapt the wall construction for the different cooling mechanism. The corrugated wall is fabricated from high-temperature alloy without integral cooling channels, with film coolant injection occurring through discrete ports positioned upstream of the corrugation initiation station. The injected coolant film distributes around the corrugated geometry, with the three-dimensional flow patterns influencing film coverage and effectiveness in ways that are accommodated through optimized injection port placement.

Embodiments incorporating transpiration cooling employ porous wall construction that permits uniform coolant emergence across the wall surface. The corrugated geometry is fabricated from sintered powder metal or laser-drilled metallic materials that provide the required porosity while maintaining structural capability. The local porosity varies circumferentially and axially to match the local cooling requirement distribution, with increased porosity at lobe peaks and shock impingement locations.

Embodiments incorporating radiation cooling eliminate active cooling entirely, relying on thermal radiation from the hot wall surface to space environment to maintain acceptable temperatures. These embodiments are applicable only for vacuum operation where the low ambient pressure eliminates convective losses and radiation becomes the dominant heat rejection mechanism. The corrugated wall is fabricated from refractory metals or ceramic materials capable of sustained operation at temperatures of fifteen hundred to two thousand Kelvin, with surface treatments or coatings optimized for high emissivity to maximize radiative heat rejection.

The foregoing detailed description presents the corrugated lobe propulsion system of the present invention in sufficient detail to enable practice by those skilled in the relevant engineering arts. The specific parameters, materials, and processes described represent preferred embodiments suitable for particular applications, while the broader inventive concepts encompass variations adapted to the diverse requirements of aerospace propulsion across the full range of vehicle types, mission profiles, and operating environments.

Abstract

A rocket propulsion system incorporating a divergent exhaust section with corrugated lobe geometry extending through approximately sixty percent of the divergent section length, measured from a nozzle exit plane and initiating at a predetermined station downstream of a throat plane. The corrugated lobe portion generates double-diamond shock patterns that progressively coalesce into single-diamond configurations in the downstream exhaust plume, while streamwise vortical structures develop from pressure gradients established by the lobe geometry to enhance mixing efficiency. The sixty-percent corrugation extent represents an optimal balance between mixing enhancement, shock management, internal dissipation losses, and boundary layer separation resistance. The system incorporates thermal management provisions addressing concentrated heating at lobe peaks, structural design accommodating three-dimensional stress distributions, and optional variable geometry mechanisms enabling adjustment of effective corrugation extent during flight. Multi-stage configurations employ differing corrugation extents optimized for atmospheric and vacuum operating regimes.

Prior Art Reference

Investigation on the Impact of Divergent Corrugated Lobe Length on Supersonic Flow
K. G. Kriparaj, P. S. Tide, and N. Biju
Journal of Spacecraft and Rockets 2025 62:6, 1882-1890

Appendix 1

The following equations establish the mathematical foundation governing the corrugated lobe propulsion system, defining the relationships between geometric parameters, flow field characteristics, and performance metrics that underpin the sixty-percent corrugation extent optimization.

Corrugated Wall Geometry Definition

$$r(z,\theta) = r_m(z) + A(z)\cos(N\theta)$$

The local wall radius at any point within the corrugated section equals the sum of the mean radius component and the circumferentially-varying perturbation component. The variable r represents the radial distance from the nozzle

centerline to the wall surface measured in meters. The axial coordinate z denotes the downstream distance from the throat plane measured in meters. The azimuthal coordinate θ represents the angular position around the nozzle circumference measured in radians from a reference meridian. The mean radius function r_m specifies the axisymmetric baseline expansion contour that would exist in the absence of corrugation. The amplitude function A defines the magnitude of the radial perturbation at each axial station. The integer N designates the number of lobes around the circumference, equal to eight for the representative implementation.

Amplitude Distribution Function

$$A(z) = A_{exit}\left[\frac{1-\cos\left(\pi\,\frac{z-z_i}{z_e-z_i}\right)}{2}\right]^{\beta}$$

The amplitude of the corrugation varies from zero at the corrugation initiation station to maximum value at the exit plane according to a modified cosine distribution. The variable A_{exit} represents the maximum amplitude achieved at the exit plane, equal to eighteen millimeters for the representative implementation. The axial position z_i denotes the corrugation initiation station location where amplitude equals zero. The exit plane position z_e marks the downstream terminus of the corrugated section. The exponent β controls the rate of amplitude growth through the corrugated section, with values greater than unity biasing the amplitude increase toward the downstream portion of the corrugation region.

Corrugation Extent Ratio

$$\xi = \frac{L_c}{L_d} = \frac{z_e - z_i}{z_e - z_t}$$

The corrugation extent ratio quantifies the fraction of the divergent section length occupied by the corrugated geometry. The parameter ξ equals the corrugation length Lc divided by the total divergent section length Ld. The corrugation length equals the difference between the exit plane position z_e and the corrugation initiation position z_i. The total divergent length equals the difference between the exit plane position z_e and the throat position z_t. For the present invention, the optimal value of the corrugation extent ratio equals zero point six, corresponding to sixty percent of the divergent section length.

Oblique Shock Wave Angle Relation

$$\tan\delta = 2\cot\sigma\left[\frac{M_1^2\sin^2\sigma - 1}{M_1^2(\gamma + \cos 2\sigma) + 2}\right]$$

The relationship between the shock wave angle and the flow deflection angle for oblique shock waves generated at lobe peaks follows from the conservation equations across the shock discontinuity. The deflection angle δ represents the angular change in flow direction caused by the lobe geometry. The shock angle σ denotes the inclination of the shock wave relative to the incoming flow direction. The upstream Mach number M_1 characterizes the supersonic flow velocity approaching the shock wave. The specific heat ratio γ equals the ratio of specific heats at constant pressure and constant volume for the combustion gas mixture, approximately one point two four for typical rocket propellants.

Pressure Ratio Across Oblique Shock

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2\sin^2\sigma - 1)$$

The static pressure increase across an oblique shock wave depends upon the upstream Mach number and the shock angle. The downstream pressure p_2 exceeds the upstream pressure p_1 by an amount determined by the normal component of the Mach number relative to the shock surface. Stronger shock waves at greater angles create larger pressure ratios, with the limiting case of a normal shock occurring when the shock angle equals ninety degrees.

Prandtl-Meyer Expansion Function

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\arctan\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \arctan\sqrt{M^2-1}$$

The Prandtl-Meyer function relates the Mach number to the turning angle through an isentropic expansion wave such as occurs at lobe trough locations. The function ν evaluated at Mach number M gives the total turning angle that the flow has undergone in expanding from sonic conditions to the specified Mach number. The difference in Prandtl-Meyer function values between two stations gives the flow turning angle through an expansion fan connecting those stations.

Circumferential Pressure Gradient

$$\frac{\partial p}{\partial \theta} = -\rho\,u^2\kappa_\theta$$

The circumferential pressure gradient that drives secondary flow and vortex generation depends upon the local flow density, velocity, and circumferential curvature of the streamlines. The density ρ represents the local mass per unit volume of the combustion gas. The velocity u denotes the local flow speed tangent to the streamline. The circumferential curvature κθ quantifies the rate of streamline turning in the circumferential direction induced by the corrugated wall geometry.

Streamwise Vorticity Generation

$$\frac{D\,\omega_z}{D\,t} = \frac{1}{\rho^2}\left(\nabla\rho\times\nabla p\right)_z + \nu\,\nabla^2\omega_z$$

The evolution of streamwise vorticity within the corrugated section follows from the vorticity transport equation applied to compressible viscous flow. The streamwise vorticity component ωz represents the rotation of fluid elements about axes parallel to the nozzle axis. The material derivative D/Dt describes the rate of change following a fluid element. The baroclinic torque term involving the cross product of density and pressure gradients generates vorticity when these gradients are not parallel, as occurs in the three-dimensional shock-expansion pattern of the corrugated section. The kinematic viscosity ν multiplying the Laplacian of vorticity represents the diffusive spreading of vorticity through viscous action.

Circulation of Individual Vortices

$$\Gamma = \oint_C \vec{u}\cdot d\,\vec{l} = \iint_S \omega_z\,d\,S$$

The circulation quantifies the strength of vortical structures through the line integral of velocity around a closed contour or equivalently the surface integral of vorticity over the enclosed area. The velocity vector u represents the local flow velocity at each point along the integration contour C. The differential path element dl follows the contour in the positive sense. The surface S spans the contour with normal direction aligned with the streamwise axis. The circulation of individual lobe-generated vortices reaches approximately twelve square meters per second at the exit plane for the representative implementation.

Mixing Enhancement Parameter

$$\eta_m = \frac{\dot{m}_{mixed}}{\dot{m}_{total}} = 1 - \exp\left(-\frac{x}{L_m}\right)$$

The mixing enhancement provided by the vortical structures is characterized by the mixed mass fraction as a function of downstream distance from the nozzle exit. The mixed mass flow rate $\dot{m}_{mixed}$ represents the portion of the total flow that has achieved thermal and compositional equilibration with the ambient or secondary fluid. The total mass flow rate $\dot{m}_{total}$ equals the sum of exhaust and entrained flows. The downstream distance x is measured from the exit plane. The mixing length L_m characterizes the axial distance required to achieve substantial mixing, with smaller values indicating more rapid mixing. The corrugated lobe geometry reduces the mixing length by approximately forty percent relative to equivalent circular nozzles.

Shock Coalescence Distance

$$x_c = C_c\cdot D_e\cdot\left(\frac{p_e}{p_a}\right)^{0.4}$$

The distance downstream of the exit plane at which the multiple shock waves from individual lobe peaks coalesce into a single unified shock structure depends upon the exit diameter, the nozzle pressure ratio, and a geometric coefficient. The coalescence distance xc is measured in the downstream direction from the exit plane. The coalescence coefficient Cc depends upon the number of lobes and the lobe amplitude ratio, with typical values between two and four for the eight-lobe configuration. The exit diameter D_e characterizes the nozzle exit dimension. The exit pressure p_e and ambient pressure p_a determine the nozzle pressure ratio that influences shock strength and interaction behavior.

Boundary Layer Momentum Thickness

$$\theta = \int_0^{\delta}\frac{\rho\,u}{\rho_e u_e}\left(1-\frac{u}{u_e}\right)d\,y$$

The momentum thickness characterizes the momentum deficit within the boundary layer relative to the freestream conditions, serving as the primary parameter for assessing boundary layer health and separation resistance. The integration proceeds from the wall surface at y equals zero through the boundary layer to the edge at y equals δ. The local density ρ and velocity u vary through the boundary layer according to the compressible boundary layer profiles. The edge density ρ_e and edge velocity u_e represent the freestream conditions immediately outside the boundary layer. Larger momentum thickness values indicate thicker, more energy-deficient boundary layers with reduced separation resistance.

Shape Factor and Separation Criterion

$$H = \frac{\delta^*}{\theta} < H_{sep}$$

The shape factor provides a sensitive indicator of boundary layer condition and separation susceptibility through the ratio of displacement thickness to momentum thickness. The displacement thickness δ* represents the distance by which the external flow is displaced outward due to the velocity deficit within the boundary layer. The momentum thickness θ quantifies the momentum deficit as defined previously. The critical shape factor H_{sep} represents the threshold value above which separation becomes imminent, approximately three point zero for turbulent boundary layers under the conditions prevailing in rocket nozzles. Maintaining the shape factor below this critical value throughout the corrugated section ensures attached flow and full performance realization.

Wall Heat Transfer

$$q_w = h(T_{aw} - T_w) = \frac{k_w}{t_w}(T_w - T_c)$$

The heat flux at the wall surface equals the convective heat transfer from the hot gas to the wall and must equal the conductive heat transfer through the wall to the coolant under steady-state conditions. The wall heat flux qw represents the thermal energy per unit area per unit time crossing the wall surface. The convective heat transfer coefficient h characterizes the effectiveness of heat transfer from the gas to the wall. The adiabatic wall temperature T_aw represents the equilibrium temperature an insulated wall would achieve. The actual wall temperature Tw results from the balance between heating and cooling. The wall thermal conductivity kw and wall thickness tw govern the conductive resistance. The coolant temperature Tc represents the heat sink condition.

Adiabatic Wall Temperature

$$T_{aw} = T_e \left[1 + r\,\frac{\gamma - 1}{2} M_e^2\right]$$

The adiabatic wall temperature exceeds the static temperature due to recovery of kinetic energy within the boundary layer. The edge static temperature T_e represents the temperature of the freestream flow outside the boundary layer. The recovery factor r characterizes the fraction of kinetic energy recovered at the wall, approximately zero point nine for turbulent boundary layers. The specific heat ratio γ and edge Mach number M_e complete the determination of the temperature recovery.

Coolant Heat Transfer Coefficient

$$h_c = \frac{Nu \cdot k_c}{D_h} = 0.023\,Re^{0.8} Pr^{0.4} \frac{k_c}{D_h}$$

The convective heat transfer coefficient on the coolant side of the wall determines the cooling effectiveness for given coolant conditions and channel geometry. The Nusselt number Nu represents the dimensionless heat transfer coefficient. The coolant thermal conductivity kc characterizes the coolant material thermal transport property. The hydraulic diameter D_h equals four times the channel cross-sectional area divided by the wetted perimeter. The Reynolds number Re characterizes the coolant flow velocity and viscosity conditions. The Prandtl number Pr represents the ratio of momentum diffusivity to thermal diffusivity for the coolant. The coefficient zero point zero two three and the exponents zero point eight and zero point four derive from the Dittus-Boelter correlation for turbulent pipe flow.

Structural Hoop Stress

$$\sigma_\theta = \frac{p \cdot r}{\rho_c \cdot t}$$

The hoop stress in the nozzle wall resists the tendency of internal pressure to expand the structure radially. The circumferential stress σθ acts tangentially around the nozzle circumference. The internal pressure p represents the local combustion gas pressure at the station of interest. The local radius r characterizes the distance from the nozzle axis to the wall. The local radius of curvature ρc in the circumferential direction modifies the stress distribution for non-circular cross-sections, with stress increasing where curvature is tight and decreasing where curvature is gentle. The wall thickness t provides the structural resistance to the pressure loading.

Thermal Stress Through Wall Thickness

$$\sigma_{thermal} = \frac{E\alpha\Delta T}{2(1-\nu)}$$

The thermal stress arising from temperature gradients through the wall thickness creates bending moments that add to the membrane stress from pressure loading. The elastic modulus E characterizes the material stiffness. The coefficient of thermal expansion α represents the fractional dimensional change per unit temperature change. The temperature difference ΔT equals the difference between hot-side and cold-side wall temperatures. The Poisson ratio ν accounts for the biaxial constraint in the thin shell geometry. The factor of two in the denominator reflects the linear temperature distribution assumption.

Thrust Coefficient

$$C_F = \sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1 - \left(\frac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(\frac{p_e - p_a}{p_c}\right)\frac{A_e}{A_t}$$

The thrust coefficient characterizes the nozzle effectiveness in converting chamber pressure into thrust, providing the primary performance metric for comparing nozzle designs. The first term under the radical represents the momentum thrust contribution arising from acceleration of the exhaust gases. The second term represents the pressure thrust contribution arising from the difference between exit pressure and ambient pressure acting over the exit area. The chamber pressure pc serves as the reference pressure. The exit pressure p_e and ambient pressure p_a determine the pressure thrust contribution. The exit area A_e and throat area A_t define the nozzle area ratio. The specific heat ratio γ governs the isentropic expansion relationships.

Specific Impulse

$$I_{sp} = \frac{F}{\dot{m}\,g_0} = \frac{C_F \cdot p_c \cdot A_t}{\dot{m}\,g_0}$$

The specific impulse represents the fundamental measure of propellant utilization efficiency, expressing the thrust produced per unit weight flow rate of propellant consumed. The thrust F equals the product of thrust coefficient, chamber pressure, and throat area. The mass flow rate ṁ represents the propellant consumption rate. The standard gravitational acceleration g_0 equals nine point eight zero six five meters per second squared, providing the conversion between mass and weight flow rates. Higher specific impulse indicates more efficient utilization of propellant mass.

Nozzle Efficiency

$$\eta_n = \frac{C_{F,actual}}{C_{F,ideal}} = 1 - \eta_{div} - \eta_{bl} - \eta_{shock}$$

The nozzle efficiency quantifies the performance achieved relative to the ideal isentropic expansion, accounting for all loss mechanisms present in the actual flow. The actual thrust coefficient CF,actual reflects the integrated effect of all flow phenomena. The ideal thrust coefficient CF,ideal represents the performance achievable with perfect isentropic expansion to the actual exit pressure. The divergence loss ηdiv accounts for the non-axial direction of flow at the exit plane. The boundary layer loss ηbl represents the momentum deficit within the viscous boundary layer. The shock loss ηshock accounts for the entropy increase through internal shock waves. For the corrugated lobe configuration, the shock loss is managed through the coalescence behavior such that overall efficiency remains within approximately one percent of equivalent conventional nozzle designs while providing the substantial mixing and acoustic benefits.

Vortex Core Tangential Velocity

$$u_\theta(r) = \frac{\Gamma}{2\pi r}\left[1 - \exp\left(-\frac{r^2}{r_c^2}\right)\right]$$

The tangential velocity distribution within and around each streamwise vortex follows the Lamb-Oseen vortex model that accounts for viscous diffusion of the vortex core. The tangential velocity uθ at radial distance r from the vortex axis depends upon the vortex circulation Γ and the core radius rc. At large distances from the vortex axis, the velocity approaches the inviscid point vortex solution proportional to the inverse of radius. Within the vortex core, the velocity increases approximately linearly with radius, reaching maximum value at approximately one point one two times the core radius. The core radius grows with downstream distance due to viscous diffusion according to the relationship rc equals the square root of four times kinematic viscosity times time.

Lobe Penetration Parameter

$$\lambda = \frac{A_{exit}}{r_{m,exit}}$$

The lobe penetration parameter characterizes the geometric intensity of the corrugation through the ratio of lobe amplitude to mean radius at the exit plane. This dimensionless parameter determines the strength of the circumferential pressure variations and the resulting vortex circulation. For the representative implementation with amplitude of eighteen millimeters and mean exit radius of one hundred sixty-two millimeters, the penetration parameter equals approximately zero point one one. Values substantially below this range produce insufficient vortex strength for effective mixing enhancement, while values substantially above this range create excessive flow disturbance with associated separation risk and efficiency penalty.

Reynolds Number at Corrugation Initiation

$$Re_i = \frac{\rho_i u_i L_c}{\mu_i}$$

The Reynolds number at the corrugation initiation station characterizes the ratio of inertial to viscous forces that governs boundary layer behavior and turbulence characteristics within the corrugated section. The density ρ_i, velocity u_i, and dynamic viscosity μ_i are evaluated at local conditions at the corrugation initiation station. The corrugation length Lc serves as the characteristic length scale. For the representative implementation, this Reynolds number exceeds ten million, ensuring fully turbulent boundary layer conditions throughout the corrugated section and enabling the boundary layer to resist separation under the three-dimensional pressure gradients imposed by the lobe geometry.

Pressure Recovery Coefficient

$$C_p = \frac{p - p_i}{\frac{1}{2}\rho_i u_i^2}$$

The pressure recovery coefficient quantifies the local pressure change relative to the dynamic pressure at the corrugation initiation station, providing a normalized measure of the pressure loading experienced by the boundary layer. The local static pressure p at any point within the corrugated section is compared to the reference pressure p_i at the corrugation initiation station. Positive values of the pressure coefficient indicate pressure rise relative to the initiation station, which would represent adverse pressure gradient for the boundary layer. The supersonic expansion through the corrugated section produces predominantly negative pressure coefficient values, representing the favorable pressure gradient that maintains boundary layer attachment despite the three-dimensional geometric perturbations.

Effective Area Ratio

$$\epsilon_{eff} = \frac{A_e - A_{bl}}{A_t} = \epsilon_{geo}\left(1 - \frac{2\delta^*}{D_e}\right)$$

The effective area ratio accounts for the boundary layer displacement effect that reduces the cross-sectional area available for the inviscid core flow. The geometric area ratio εgeo equals the physical exit area A_e divided by the throat area A_t. The displacement thickness δ* at the exit plane reduces the effective flow area by an annular region of thickness δ* around the circumference. For exit diameter D_e, this correction reduces the effective area ratio by the factor in parentheses. Accurate prediction of nozzle performance requires use of the effective area ratio rather than the geometric area ratio in the thrust coefficient and specific impulse calculations.

Acoustic Power Spectral Density

$$S_{pp}(f) = \frac{\rho_\infty^2 c_\infty^4}{r^2}\int_V \left|\hat{T}_{ij}(f)\right|^2 dV$$

The acoustic power spectral density characterizes the distribution of noise energy across frequency, enabling quantification of the acoustic benefits provided by the corrugated lobe geometry. The spectral density S_{pp} at frequency f depends upon the ambient density $\rho\infty$ and speed of sound $c\infty$, the observation distance r, and the volume integral of the Lighthill stress tensor Fourier transform magnitude squared. The corrugated lobe geometry modifies the stress tensor distribution through the altered shock structure and vortical mixing patterns, reducing the high-frequency content associated with small-scale shock oscillations while introducing low-frequency content associated with the organized vortex structures.

Overall Sound Pressure Level

$$OASPL = 10\log_{10}\left(\frac{\int_0^\infty S_{pp}(f)df}{p_{ref}^2}\right)$$

The overall sound pressure level integrates the power spectral density across all frequencies to provide a single measure of total acoustic emission intensity. The reference pressure p_{ref} equals twenty micropascals, the standard reference for sound pressure level measurements in air. The logarithmic scale expresses the result in decibels. The corrugated lobe geometry achieves overall sound pressure level reduction of approximately three decibels relative to equivalent conventional nozzles, corresponding to halving of the acoustic power emission and significant reduction in noise impact on surrounding structures and personnel.

Thermal Effectiveness

$$\epsilon_{th} = \frac{T_{w,max} - T_{c,in}}{T_{aw} - T_{c,in}}$$

The thermal effectiveness characterizes the cooling system performance through the ratio of achieved temperature rise to maximum possible temperature rise. The maximum wall temperature Tw,max represents the hottest point on the wall surface, typically occurring at lobe peak locations where convective heating is most intense. The coolant inlet temperature Tc,in represents the heat sink condition. The adiabatic wall temperature T_aw represents the theoretical maximum wall temperature in the absence of cooling. Lower values of thermal effectiveness indicate more effective cooling with greater margin between actual wall temperature and the adiabatic limit. For the representative implementation, thermal effectiveness equals approximately zero point four five at the most critical lobe peak location, providing adequate margin for reliable operation.

Coolant Pressure Drop

$$\Delta p_c = f\frac{L_c}{D_h}\frac{\rho_c u_c^2}{2} + K_{ent}\frac{\rho_c u_c^2}{2} + K_{exit}\frac{\rho_c u_c^2}{2}$$

The pressure drop through the cooling circuit comprises frictional losses along the channel length and minor losses at the entrance and exit manifolds. The friction factor f depends upon the channel Reynolds number and surface roughness. The channel length Lc and hydraulic diameter D_h determine the friction loss magnitude. The entrance and exit loss coefficients K_{ent} and K_{exit} characterize the minor losses at the manifold interfaces. The coolant density ρc and velocity uc establish the dynamic pressure that scales all loss terms. For the representative implementation, total cooling circuit pressure drop equals approximately four point three bar, requiring corresponding margin in the propellant feed system pressure capability.

Fatigue Life Estimation

$$N_f = \left(\frac{\sigma_a}{A}\right)^{-\frac{1}{b}}$$

The fatigue life in cycles to failure depends upon the alternating stress amplitude and the material fatigue properties. The alternating stress σ_a equals one-half the stress range from minimum to maximum during each loading cycle. The coefficient A and exponent b are material constants determined from fatigue testing, with typical values for Inconel 718 at operating temperature of A equals one thousand two hundred megapascals and b equals zero point one two. For the representative implementation with alternating stress of sixty-five megapascals, the predicted fatigue life exceeds ten thousand cycles, substantially exceeding the one hundred cycle requirement with appropriate safety factor.

Manufacturing Tolerance Stack-Up

$$\Delta_{total} = \sqrt{\sum_{i=1}^{n}\Delta_i^2}$$

The total dimensional variation resulting from multiple independent tolerance contributions follows the root-sum-square combination rule for statistically independent variations. Each individual tolerance contribution Δ_i represents a single source of dimensional uncertainty such as additive manufacturing build accuracy, thermal distortion, machining tolerance, or assembly alignment. The total variation Δ_{total} characterizes the expected range of the resulting dimension. For the representative implementation, the critical throat diameter tolerance of plus or minus zero point one millimeters results from combination of build tolerance of zero point zero seven five millimeters, thermal distortion tolerance of zero point zero five millimeters, and measurement uncertainty of zero point zero three millimeters.

Appendix 2

```python
"""
Corrugated Lobe Propulsion System - Complete Implementation
A comprehensive computational framework for design, analysis, and optimization
of corrugated lobe rocket nozzle geometries with integrated thermal, structural,
and performance prediction capabilities.
"""

import numpy as np
from numpy import pi, sin, cos, tan, arctan, sqrt, exp, log, log10
from numpy.linalg import norm, solve, inv
from scipy.integrate import odeint, quad, dblquad, solve_ivp
from scipy.optimize import fsolve, minimize, brentq, newton
from scipy.interpolate import interp1d, RectBivariateSpline, CubicSpline
from scipy.special import gamma as gamma_func
from scipy.sparse import csr_matrix, lil_matrix
from scipy.sparse.linalg import spsolve
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from dataclasses import dataclass, field
from typing import List, Tuple, Optional, Callable, Dict, Any
from enum import Enum
import json
import warnings
from abc import ABC, abstractmethod
from functools import lru_cache
import multiprocessing as mp
from concurrent.futures import ProcessPoolExecutor, ThreadPoolExecutor


# ============================================================================
# Physical Constants and Unit Conversions
# ============================================================================

class PhysicalConstants:
    """Fundamental physical constants and unit conversion factors."""

    UNIVERSAL_GAS_CONSTANT = 8314.46261815324  # J/(kmol·K)
    STANDARD_GRAVITY = 9.80665  # m/s²
    STEFAN_BOLTZMANN = 5.670374419e-8  # W/(m² K⁴)
    BOLTZMANN_CONSTANT = 1.380649e-23  # J/K
    AVOGADRO_NUMBER = 6.02214076e23  # 1/mol
    REFERENCE_PRESSURE_ACOUSTIC = 20e-6  # Pa (reference for SPL)

    @staticmethod
    def bar_to_pascal(bar: float) -> float:
        return bar * 1e5

    @staticmethod
    def pascal_to_bar(pascal: float) -> float:
        return pascal / 1e5

    @staticmethod
    def celsius_to_kelvin(celsius: float) -> float:
        return celsius + 273.15

    @staticmethod
    def kelvin_to_celsius(kelvin: float) -> float:
        return kelvin - 273.15


# ============================================================================
# Material Property Database
# ============================================================================

@dataclass
class MaterialProperties:
```

```
    """Complete material property specification for structural analysis."""

    name: str
    density: float  # kg/m³
    elastic_modulus: float  # Pa
    poisson_ratio: float
    yield_strength: float  # Pa
    ultimate_strength: float  # Pa
    thermal_conductivity: float  # W/(m·K)
    specific_heat: float  # J/(kg·K)
    thermal_expansion: float  # 1/K
    melting_point: float  # K
    max_service_temperature: float  # K
    fatigue_coefficient: float  # Pa
    fatigue_exponent: float

    def get_elastic_modulus_at_temperature(self, temperature: float) -> float:
        """Temperature-dependent elastic modulus with polynomial correction."""
        ratio = temperature / self.max_service_temperature
        correction = 1.0 - 0.3 * ratio - 0.15 * ratio**2
        return self.elastic_modulus * max(correction, 0.4)

    def get_yield_strength_at_temperature(self, temperature: float) -> float:
        """Temperature-dependent yield strength."""
        ratio = temperature / self.max_service_temperature
        correction = 1.0 - 0.4 * ratio - 0.2 * ratio**2
        return self.yield_strength * max(correction, 0.3)

    def get_thermal_conductivity_at_temperature(self, temperature: float) -> float:
        """Temperature-dependent thermal conductivity."""
        ratio = temperature / 1000.0
        return self.thermal_conductivity * (1.0 + 0.15 * ratio)


class MaterialDatabase:
    """Database of common aerospace materials with complete property sets."""

    INCONEL_718 = MaterialProperties(
        name="Inconel 718",
        density=8190.0,
        elastic_modulus=200e9,
        poisson_ratio=0.29,
        yield_strength=1034e6,
        ultimate_strength=1241e6,
        thermal_conductivity=11.4,
        specific_heat=435.0,
        thermal_expansion=13.0e-6,
        melting_point=1609.0,
        max_service_temperature=973.0,
        fatigue_coefficient=1200e6,
        fatigue_exponent=0.12
    )

    COPPER_ALLOY_CUCRZR = MaterialProperties(
        name="CuCrZr",
        density=8900.0,
        elastic_modulus=130e9,
        poisson_ratio=0.34,
        yield_strength=450e6,
        ultimate_strength=520e6,
        thermal_conductivity=320.0,
        specific_heat=385.0,
        thermal_expansion=17.0e-6,
        melting_point=1356.0,
        max_service_temperature=723.0,
        fatigue_coefficient=800e6,
        fatigue_exponent=0.10
    )

    NIOBIUM_C103 = MaterialProperties(
        name="Niobium C103",
        density=8860.0,
        elastic_modulus=100e9,
        poisson_ratio=0.38,
        yield_strength=275e6,
        ultimate_strength=390e6,
        thermal_conductivity=43.0,
        specific_heat=272.0,
        thermal_expansion=7.2e-6,
        melting_point=2741.0,
        max_service_temperature=1473.0,
        fatigue_coefficient=600e6,
        fatigue_exponent=0.14
    )

    RHENIUM = MaterialProperties(
        name="Rhenium",
        density=21020.0,
        elastic_modulus=463e9,
        poisson_ratio=0.30,
        yield_strength=290e6,
        ultimate_strength=1070e6,
        thermal_conductivity=48.0,
        specific_heat=137.0,
        thermal_expansion=6.2e-6,
        melting_point=3459.0,
        max_service_temperature=2273.0,
        fatigue_coefficient=900e6,
        fatigue_exponent=0.11
    )


# ============================================================================
# Propellant Combustion Properties
# ============================================================================

@dataclass
class PropellantCombination:
    """Complete propellant combustion property specification."""

    name: str
    oxidizer: str
    fuel: str
    mixture_ratio: float  # O/F mass ratio
    molecular_weight: float  # kg/kmol
    specific_heat_ratio: float
    characteristic_velocity: float  # m/s
    combustion_temperature: float  # K
    specific_heat_constant_pressure: float  # J/(kg·K)
    viscosity_reference: float  # Pa·s at reference temperature
    viscosity_temperature_exponent: float
    thermal_conductivity_gas: float  # W/(m·K)
    prandtl_number: float

    def get_gas_constant(self) -> float:
        """Specific gas constant for the combustion products."""
        return PhysicalConstants.UNIVERSAL_GAS_CONSTANT / self.molecular_weight

    def get_viscosity(self, temperature: float) -> float:
        """Temperature-dependent dynamic viscosity using power law."""
        T_ref = 3000.0
        return self.viscosity_reference * (temperature / T_ref) ** self.viscosity_temperature_exponent

    def get_thermal_conductivity(self, temperature: float) -> float:
        """Temperature-dependent thermal conductivity."""
        T_ref = 3000.0
        return self.thermal_conductivity_gas * (temperature / T_ref) ** 0.7

    def get_density(self, pressure: float, temperature: float) -> float:
        """Ideal gas density calculation."""
        R = self.get_gas_constant()
        return pressure / (R * temperature)


class PropellantDatabase:
    """Database of common propellant combinations."""

    LOX_RP1 = PropellantCombination(
        name="LOX/RP-1",
        oxidizer="Liquid Oxygen",
        fuel="RP-1 Kerosene",
        mixture_ratio=2.56,
        molecular_weight=23.3,
        specific_heat_ratio=1.24,
        characteristic_velocity=1774.0,
        combustion_temperature=3670.0,
        specific_heat_constant_pressure=2010.0,
        viscosity_reference=9.5e-5,
        viscosity_temperature_exponent=0.68,
        thermal_conductivity_gas=0.18,
        prandtl_number=0.82
    )

    LOX_LH2 = PropellantCombination(
        name="LOX/LH2",
        oxidizer="Liquid Oxygen",
        fuel="Liquid Hydrogen",
        mixture_ratio=6.0,
        molecular_weight=12.0,
        specific_heat_ratio=1.26,
        characteristic_velocity=2386.0,
        combustion_temperature=3520.0,
        specific_heat_constant_pressure=4120.0,
        viscosity_reference=7.2e-5,
        viscosity_temperature_exponent=0.72,
        thermal_conductivity_gas=0.42,
        prandtl_number=0.71
    )

    NTO_MMH = PropellantCombination(
        name="NTO/MMH",
        oxidizer="Nitrogen Tetroxide",
        fuel="Monomethylhydrazine",
        mixture_ratio=1.65,
        molecular_weight=21.8,
        specific_heat_ratio=1.25,
        characteristic_velocity=1720.0,
        combustion_temperature=3250.0,
        specific_heat_constant_pressure=1850.0,
        viscosity_reference=8.8e-5,
        viscosity_temperature_exponent=0.65,
        thermal_conductivity_gas=0.15,
        prandtl_number=0.85
    )


# ============================================================================
# Corrugated Lobe Geometry Definition
# ============================================================================

@dataclass
class CorrugatedLobeParameters:
    """Complete geometric specification for the corrugated lobe nozzle."""

    # Throat geometry
    throat_radius: float  # m
    throat_radius_of_curvature_upstream: float  # m
    throat_radius_of_curvature_downstream: float  # m

    # Divergent section geometry
    divergent_half_angle_initial: float  # radians
    divergent_half_angle_exit: float  # radians
    area_ratio: float
    divergent_length: float  # m

    # Corrugation parameters
    number_of_lobes: int
    corrugation_extent_ratio: float  # fraction of divergent length
    exit_amplitude: float  # m
    amplitude_exponent: float
    lobe_phase_angle: float  # radians

    # Convergent section (optional)
    convergent_half_angle: float = 0.5236  # radians (30 degrees default)
    convergent_length: float = 0.0  # m, computed if zero
    chamber_radius: float = 0.0  # m, computed if zero
    contraction_ratio: float = 3.0

    def __post_init__(self):
        """Compute derived geometric parameters."""
        self.throat_area = pi * self.throat_radius**2
        self.exit_radius = self.throat_radius * sqrt(self.area_ratio)
        self.exit_area = pi * self.exit_radius**2

        if self.chamber_radius == 0.0:
            self.chamber_radius = self.throat_radius * sqrt(self.contraction_ratio)

        if self.convergent_length == 0.0:
            self.convergent_length = (
                (self.chamber_radius - self.throat_radius) /
                tan(self.convergent_half_angle)
            )

        # Corrugation initiation position
        self.corrugation_start_position = (
            self.divergent_length * (1.0 - self.corrugation_extent_ratio)
        )

        # Corrugation length
        self.corrugation_length = (
            self.divergent_length * self.corrugation_extent_ratio
        )

        # Lobe penetration parameter
        self.penetration_parameter = self.exit_amplitude / self.exit_radius


class NozzleContourGenerator:
    """Generates the complete three-dimensional nozzle contour."""

    def __init__(self, params: CorrugatedLobeParameters):
        self.params = params
        self._build_axisymmetric_contour()

    def _build_axisymmetric_contour(self):
        """Construct the baseline axisymmetric contour before corrugation."""
        p = self.params

        # Number of axial stations for contour definition
        n_stations = 500

        # Convergent section
        z_conv = np.linspace(-p.convergent_length, 0, n_stations // 4)
        r_conv = np.zeros_like(z_conv)

        for i, z in enumerate(z_conv):
            # Parabolic convergent contour
            z_norm = (z + p.convergent_length) / p.convergent_length
            r_conv[i] = p.chamber_radius - (p.chamber_radius - p.throat_radius) * z_norm**1.5

        # Throat region with circular arc
        theta_range = np.linspace(-pi/4, pi/4, n_stations // 4)
        z_throat = p.throat_radius_of_curvature_downstream * np.sin(theta_range)
        r_throat = (
            p.throat_radius +
            p.throat_radius_of_curvature_downstream * (1 - np.cos(theta_range))
        )

        # Divergent section using Rao optimum contour approximation
        z_div = np.linspace(
            z_throat[-1] + 0.001,
            p.divergent_length,
            n_stations // 2
        )
        r_div = np.zeros_like(z_div)

        # Initial expansion angle
        theta_init = p.divergent_half_angle_initial
        # Exit angle
        theta_exit = p.divergent_half_angle_exit

        for i, z in enumerate(z_div):
            z_norm = z / p.divergent_length
            # Parabolic angle variation (Rao contour approximation)
            local_angle = theta_init - (theta_init - theta_exit) * z_norm**0.8

            if i == 0:
                r_div[i] = r_throat[-1] + z_div[0] * tan(theta_init)
            else:
                dz = z_div[i] - z_div[i-1]
                r_div[i] = r_div[i-1] + dz * tan(local_angle)

        # Combine sections
        self.z_contour = np.concatenate([z_conv, z_throat[1:], z_div])
        self.r_contour = np.concatenate([r_conv, r_throat[1:], r_div])

        # Create interpolation function
        self.r_mean_interp = interp1d(
            self.z_contour,
            self.r_contour,
            kind='cubic',
            fill_value='extrapolate'
        )

    def get_mean_radius(self, z: float) -> float:
        """Get the axisymmetric mean radius at axial position z."""
        return float(self.r_mean_interp(z))

    def get_amplitude(self, z: float) -> float:
        """Get the corrugation amplitude at axial position z."""
        p = self.params

        if z < p.corrugation_start_position:
            return 0.0

        z_local = z - p.corrugation_start_position
        z_norm = z_local / p.corrugation_length

        # Modified cosine distribution with exponent
        amplitude = p.exit_amplitude * (
            (1.0 - cos(pi * z_norm)) / 2.0
        ) ** p.amplitude_exponent

        return amplitude

    def get_radius(self, z: float, theta: float) -> float:
        """Get the wall radius at position (z, θ)."""
        r_mean = self.get_mean_radius(z)
        amplitude = self.get_amplitude(z)

        N = self.params.number_of_lobes
        phi = self.params.lobe_phase_angle

        r = r_mean + amplitude * cos(N * theta + phi)

        return r

    def get_surface_point(self, z: float, theta: float) -> np.ndarray:
        """Get Cartesian coordinates of a surface point."""
        r = self.get_radius(z, theta)
        x = r * cos(theta)
        y = r * sin(theta)
        return np.array([x, y, z])

    def get_surface_normal(self, z: float, theta: float) -> np.ndarray:
        """Compute the outward unit normal vector at a surface point."""
```

```
        # Numerical derivatives
        dz = 1e-6
        dtheta = 1e-6

        p0 = self.get_surface_point(z, theta)
        pz = self.get_surface_point(z + dz, theta)
        pt = self.get_surface_point(z, theta + dtheta)

        # Tangent vectors
        t_z = (pz - p0) / dz
        t_theta = (pt - p0) / dtheta

        # Normal vector (outward)
        n = np.cross(t_z, t_theta)
        n = n / norm(n)

        # Ensure outward direction
        r_vec = np.array([p0[0], p0[1], 0])
        if np.dot(n[:2], r_vec[:2]) < 0:
            n = -n

        return n

    def get_local_curvature(self, z: float, theta: float) -> Tuple[float, float]:
        """Compute principal curvatures at a surface point."""
        # Numerical computation of curvature tensor
        h = 1e-5

        # Get neighboring points for numerical differentiation
        p_zp = self.get_surface_point(z + h, theta)
        p_zm = self.get_surface_point(z - h, theta)
        p_tp = self.get_surface_point(z, theta + h)
        p_tm = self.get_surface_point(z, theta - h)
        p_0 = self.get_surface_point(z, theta)

        # First derivatives
        r_z = (p_zp - p_zm) / (2 * h)
        r_t = (p_tp - p_tm) / (2 * h)

        # Second derivatives
        r_zz = (p_zp - 2*p_0 + p_zm) / h**2
        r_tt = (p_tp - 2*p_0 + p_tm) / h**2

        p_zp_tp = self.get_surface_point(z + h, theta + h)
        p_zm_tm = self.get_surface_point(z - h, theta - h)
        p_zp_tm = self.get_surface_point(z + h, theta - h)
        p_zm_tp = self.get_surface_point(z - h, theta + h)
        r_zt = (p_zp_tp - p_zp_tm - p_zm_tp + p_zm_tm) / (4 * h**2)

        # First fundamental form coefficients
        E = np.dot(r_z, r_z)
        F = np.dot(r_z, r_t)
        G = np.dot(r_t, r_t)

        # Normal vector
        n = np.cross(r_z, r_t)
        n = n / norm(n)

        # Second fundamental form coefficients
        L = np.dot(r_zz, n)
        M = np.dot(r_zt, n)
        N_coef = np.dot(r_tt, n)

        # Principal curvatures from eigenvalue problem
        discriminant = (E*N_coef - 2*F*M + G*L)**2 - 4*(E*G - F**2)*(L*N_coef - M**2)
        if discriminant < 0:
            discriminant = 0

        H = (E*N_coef - 2*F*M + G*L) / (2*(E*G - F**2) + 1e-12)
        K = (L*N_coef - M**2) / (E*G - F**2 + 1e-12)

        kappa_1 = H + sqrt(max(H**2 - K, 0))
        kappa_2 = H - sqrt(max(H**2 - K, 0))

        return kappa_1, kappa_2

    def generate_surface_mesh(
        self,
        n_axial: int = 200,
        n_circumferential: int = 180
    ) -> Tuple[np.ndarray, np.ndarray, np.ndarray]:
        """Generate a complete surface mesh of the nozzle."""
        p = self.params

        z_vals = np.linspace(-p.convergent_length, p.divergent_length, n_axial)
        theta_vals = np.linspace(0, 2*pi, n_circumferential)

        X = np.zeros((n_axial, n_circumferential))
        Y = np.zeros((n_axial, n_circumferential))
        Z = np.zeros((n_axial, n_circumferential))

        for i, z in enumerate(z_vals):
            for j, theta in enumerate(theta_vals):
                point = self.get_surface_point(z, theta)
                X[i, j] = point[0]
                Y[i, j] = point[1]
                Z[i, j] = point[2]

        return X, Y, Z

    def get_cross_section_area(self, z: float, n_points: int = 360) -> float:
        """Compute the cross-sectional area at axial position z."""
        theta_vals = np.linspace(0, 2*pi, n_points + 1)[:-1]
        dtheta = 2*pi / n_points

        area = 0.0
        for theta in theta_vals:
            r = self.get_radius(z, theta)
            area += 0.5 * r**2 * dtheta

        return area

    def get_cross_section_perimeter(self, z: float, n_points: int = 360) -> float:
        """Compute the cross-sectional perimeter at axial position z."""
        theta_vals = np.linspace(0, 2*pi, n_points + 1)

        perimeter = 0.0
        for i in range(n_points):
            p1 = self.get_surface_point(z, theta_vals[i])[:2]
            p2 = self.get_surface_point(z, theta_vals[i+1])[:2]
            perimeter += norm(p2 - p1)

        return perimeter

    def get_hydraulic_diameter(self, z: float) -> float:
        """Compute the hydraulic diameter at axial position z."""
        area = self.get_cross_section_area(z)
        perimeter = self.get_cross_section_perimeter(z)
        return 4.0 * area / perimeter


# ============================================================================
# Compressible Flow Relations
# ============================================================================

class CompressibleFlowRelations:
    """Isentropic and shock relations for compressible gas dynamics."""

    def __init__(self, gamma: float):
        self.gamma = gamma
        self.g = gamma
        self.gp1 = gamma + 1
        self.gm1 = gamma - 1

    def area_mach_relation(self, mach: float) -> float:
        """Area ratio as function of Mach number (A/A*)."""
        M = mach
        g = self.gamma

        term1 = 2.0 / (g + 1)
        term2 = 1.0 + (g - 1) / 2.0 * M**2
        exponent = (g + 1) / (2 * (g - 1))

        area_ratio = (1.0 / M) * (term1 * term2) ** exponent
        return area_ratio

    def mach_from_area_ratio(
        self,
        area_ratio: float,
        supersonic: bool = True
    ) -> float:
        """Compute Mach number from area ratio."""

        def equation(M):
            return self.area_mach_relation(M) - area_ratio

        if supersonic:
            M_guess = 2.0
            M_bounds = (1.0001, 20.0)
        else:
            M_guess = 0.5
            M_bounds = (0.001, 0.9999)

        try:
            result = brentq(equation, M_bounds[0], M_bounds[1])
        except ValueError:
            result = newton(equation, M_guess)

        return result

    def pressure_ratio(self, mach: float) -> float:
        """Static to total pressure ratio p/p0."""
        g = self.gamma
        return (1.0 + (g - 1) / 2.0 * mach**2) ** (-g / (g - 1))

    def temperature_ratio(self, mach: float) -> float:
        """Static to total temperature ratio T/T0."""
        g = self.gamma
        return (1.0 + (g - 1) / 2.0 * mach**2) ** (-1)

    def density_ratio(self, mach: float) -> float:
        """Static to total density ratio rho/rho0."""
        g = self.gamma
        return (1.0 + (g - 1) / 2.0 * mach**2) ** (-1 / (g - 1))

    def prandtl_meyer_function(self, mach: float) -> float:
        """Prandtl-Meyer expansion function nu(M) in radians."""
        if mach <= 1.0:
            return 0.0

        g = self.gamma
        term1 = sqrt((g + 1) / (g - 1))
        term2 = arctan(sqrt((g - 1) / (g + 1) * (mach**2 - 1)))
        term3 = arctan(sqrt(mach**2 - 1))

        return term1 * term2 - term3

    def mach_from_prandtl_meyer(self, nu: float) -> float:
        """Inverse Prandtl-Meyer function to get Mach from turning angle."""

        def equation(M):
            return self.prandtl_meyer_function(M) - nu

        # Newton-Raphson with numerical derivative
        M = 2.0  # Initial guess
        for _ in range(50):
            f = equation(M)
            if abs(f) < 1e-10:
                break

            h = 1e-6
            df = (equation(M + h) - equation(M - h)) / (2 * h)
            M = M - f / df
            M = max(M, 1.001)

        return M

    def oblique_shock_angle(
        self,
        mach: float,
        deflection: float,
        weak: bool = True
    ) -> float:
        """Compute oblique shock wave angle from upstream Mach and deflection."""
        g = self.gamma
        M = mach
        delta = deflection

        def equation(sigma):
            if sigma <= delta or sigma >= pi/2:
                return 1e10

            tan_delta = tan(delta)
            M_sin = M * sin(sigma)

            if M_sin <= 1.0:
                return 1e10

            numerator = M**2 * sin(sigma)**2 - 1
            denominator = M**2 * (g + cos(2*sigma)) + 2

            return 2 * (1/tan(sigma)) * numerator / denominator - tan_delta

        # Search for solution
        if weak:
            sigma_range = np.linspace(delta + 0.01, pi/2 - 0.01, 1000)
        else:
            sigma_range = np.linspace(pi/2 - 0.01, delta + 0.01, 1000)

        for i in range(len(sigma_range) - 1):
            if equation(sigma_range[i]) * equation(sigma_range[i+1]) < 0:
                sigma = brentq(equation, sigma_range[i], sigma_range[i+1])
                return sigma

        # Fallback to approximate relation
        return delta + arcsin(1/M)

    def normal_shock_pressure_ratio(self, mach: float) -> float:
        """Pressure ratio across normal shock p2/p1."""
        g = self.gamma
        M = mach
        return 1 + 2*g/(g+1) * (M**2 - 1)

    def normal_shock_downstream_mach(self, mach: float) -> float:
        """Downstream Mach number after normal shock."""
        g = self.gamma
        M = mach

        M2_squared = (1 + (g-1)/2 * M**2) / (g * M**2 - (g-1)/2)
        return sqrt(M2_squared)

    def oblique_shock_pressure_ratio(self, mach: float, sigma: float) -> float:
        """Pressure ratio across oblique shock."""
        M_n = mach * sin(sigma)
        return self.normal_shock_pressure_ratio(M_n)

    def speed_of_sound(self, temperature: float, R: float) -> float:
        """Speed of sound from temperature and gas constant."""
        return sqrt(self.gamma * R * temperature)


# ============================================================================
# Flow Field Solver
# ============================================================================

class FlowFieldSolver:
    """Quasi-one-dimensional flow solver with three-dimensional corrections."""

    def __init__(
        self,
        geometry: NozzleContourGenerator,
        propellant: PropellantCombination,
        chamber_pressure: float,
        ambient_pressure: float
    ):
        self.geometry = geometry
        self.propellant = propellant
        self.p_c = chamber_pressure
        self.p_a = ambient_pressure

        self.gas_relations = CompressibleFlowRelations(propellant.specific_heat_ratio)
        self.R = propellant.get_gas_constant()
        self.T_c = propellant.combustion_temperature

        self._solve_quasi_1d_flow()

    def _solve_quasi_1d_flow(self):
        """Solve the quasi-one-dimensional isentropic flow field."""
        p = self.geometry.params

        # Create axial grid
        n_stations = 500
        self.z_stations = np.linspace(-p.convergent_length, p.divergent_length, n_stations)

        # Initialize flow property arrays
        self.mach = np.zeros(n_stations)
        self.pressure = np.zeros(n_stations)
        self.temperature = np.zeros(n_stations)
        self.density = np.zeros(n_stations)
        self.velocity = np.zeros(n_stations)
        self.area = np.zeros(n_stations)

        A_throat = p.throat_area
        gamma = self.propellant.specific_heat_ratio

        for i, z in enumerate(self.z_stations):
            A = self.geometry.get_cross_section_area(z)
            self.area[i] = A

            area_ratio = A / A_throat

            if z < 0:
                # Subsonic convergent section
                M = self.gas_relations.mach_from_area_ratio(area_ratio, supersonic=False)
            elif z == 0:
                M = 1.0
            else:
                # Supersonic divergent section
                M = self.gas_relations.mach_from_area_ratio(area_ratio, supersonic=True)

            self.mach[i] = M

            # Isentropic relations
            T_ratio = self.gas_relations.temperature_ratio(M)
            p_ratio = self.gas_relations.pressure_ratio(M)
            rho_ratio = self.gas_relations.density_ratio(M)

            self.temperature[i] = self.T_c * T_ratio
            self.pressure[i] = self.p_c * p_ratio
            self.density[i] = self.propellant.get_density(self.p_c, self.T_c) * rho_ratio

            a = self.gas_relations.speed_of_sound(self.temperature[i], self.R)
            self.velocity[i] = M * a

        # Exit conditions
        self.mach_exit = self.mach[-1]
```

```
        self.pressure_exit = self.pressure[-1]
        self.temperature_exit = self.temperature[-1]
        self.density_exit = self.density[-1]
        self.velocity_exit = self.velocity[-1]

        # Create interpolation functions
        self._create_interpolators()

    def _create_interpolators(self):
        """Create interpolation functions for flow properties."""
        self.mach_interp = interp1d(self.z_stations, self.mach, kind='cubic')
        self.pressure_interp = interp1d(self.z_stations, self.pressure, kind='cubic')
        self.temperature_interp = interp1d(self.z_stations, self.temperature, kind='cubic')
        self.density_interp = interp1d(self.z_stations, self.density, kind='cubic')
        self.velocity_interp = interp1d(self.z_stations, self.velocity, kind='cubic')

    def get_mach(self, z: float) -> float:
        """Get Mach number at axial position z."""
        return float(self.mach_interp(z))

    def get_pressure(self, z: float) -> float:
        """Get static pressure at axial position z."""
        return float(self.pressure_interp(z))

    def get_temperature(self, z: float) -> float:
        """Get static temperature at axial position z."""
        return float(self.temperature_interp(z))

    def get_density(self, z: float) -> float:
        """Get density at axial position z."""
        return float(self.density_interp(z))

    def get_velocity(self, z: float) -> float:
        """Get flow velocity at axial position z."""
        return float(self.velocity_interp(z))

    def compute_3d_pressure_field(
        self,
        z: float,
        theta: float
    ) -> float:
        """Compute local pressure including three-dimensional effects."""
        p = self.geometry.params

        if z < p.corrugation_start_position:
            return self.get_pressure(z)

        # Base pressure from quasi-1D solution
        p_base = self.get_pressure(z)
        M = self.get_mach(z)
        gamma = self.propellant.specific_heat_ratio

        # Local wall curvature effect
        r_mean = self.geometry.get_mean_radius(z)
        amplitude = self.geometry.get_amplitude(z)
        N = p.number_of_lobes
        phi = p.lobe_phase_angle

        # Circumferential variation due to lobe geometry
        # Pressure increases at troughs (convex wall), decreases at peaks (concave wall)
        curvature_perturbation = amplitude * N**2 * cos(N * theta + phi) / r_mean**2

        # Pressure perturbation from centrifugal effects
        rho = self.get_density(z)
        u = self.get_velocity(z)

        delta_p = -rho * u**2 * curvature_perturbation / (1 + (gamma - 1)/2 * M**2)

        return p_base + delta_p

    def compute_circumferential_velocity(
        self,
        z: float,
        theta: float
    ) -> float:
        """Compute circumferential velocity component induced by lobe geometry."""
        p = self.geometry.params

        if z < p.corrugation_start_position:
            return 0.0

        M = self.get_mach(z)
        u = self.get_velocity(z)

        amplitude = self.geometry.get_amplitude(z)
        r_mean = self.geometry.get_mean_radius(z)
        N = p.number_of_lobes
        phi = p.lobe_phase_angle

        # Estimate circumferential velocity from pressure gradient
        # and momentum conservation
        v_theta = u * amplitude * N * sin(N * theta + phi) / r_mean

        return v_theta

    def compute_mass_flow_rate(self) -> float:
        """Compute the mass flow rate through the nozzle."""
        p = self.geometry.params
        gamma = self.propellant.specific_heat_ratio

        # Critical mass flux
        m_dot = (
            p.throat_area * self.p_c /
            sqrt(self.R * self.T_c) *
            sqrt(gamma) *
            (2 / (gamma + 1)) ** ((gamma + 1) / (2 * (gamma - 1)))
        )

        return m_dot

    def compute_thrust(self) -> float:
        """Compute the thrust produced by the nozzle."""
        m_dot = self.compute_mass_flow_rate()
        p = self.geometry.params

        # Momentum thrust
        F_momentum = m_dot * self.velocity_exit

        # Pressure thrust
        F_pressure = (self.pressure_exit - self.p_a) * p.exit_area

        return F_momentum + F_pressure

    def compute_thrust_coefficient(self) -> float:
        """Compute the thrust coefficient."""
        F = self.compute_thrust()
        p = self.geometry.params

        return F / (self.p_c * p.throat_area)

    def compute_specific_impulse(self) -> float:
        """Compute the specific impulse in seconds."""
        F = self.compute_thrust()
        m_dot = self.compute_mass_flow_rate()

        return F / (m_dot * PhysicalConstants.STANDARD_GRAVITY)

    def compute_ideal_thrust_coefficient(self) -> float:
        """Compute the ideal (loss-free) thrust coefficient."""
        gamma = self.propellant.specific_heat_ratio
        p_e = self.pressure_exit
        p_a = self.p_a
        p = self.geometry.params

        term1 = 2 * gamma**2 / (gamma - 1)
        term2 = (2 / (gamma + 1)) ** ((gamma + 1) / (gamma - 1))
        term3 = 1 - (p_e / self.p_c) ** ((gamma - 1) / gamma)

        C_F_ideal = sqrt(term1 * term2 * term3)
        C_F_ideal += (p_e - p_a) / self.p_c * p.area_ratio

        return C_F_ideal

    def compute_nozzle_efficiency(self) -> float:
        """Compute the nozzle efficiency."""
        C_F_actual = self.compute_thrust_coefficient()
        C_F_ideal = self.compute_ideal_thrust_coefficient()

        return C_F_actual / C_F_ideal

# ============================================================
# Vortex Generation and Mixing Analysis
# ============================================================

class VortexAnalyzer:
    """Analysis of vortex generation and mixing enhancement."""

    def __init__(
        self,
        geometry: NozzleContourGenerator,
        flow_solver: FlowFieldSolver
    ):
        self.geometry = geometry
        self.flow = flow_solver
        self.params = geometry.params

        self._compute_vortex_characteristics()

    def _compute_vortex_characteristics(self):
        """Compute vortex strength and distribution through the corrugated section."""
        p = self.params

        n_stations = 100
        z_corr = np.linspace(
            p.corrugation_start_position,
            p.divergent_length,
            n_stations
        )

        self.z_vortex = z_corr
        self.circulation = np.zeros(n_stations)
        self.core_radius = np.zeros(n_stations)
        self.vorticity_max = np.zeros(n_stations)

        gamma = self.flow.propellant.specific_heat_ratio

        for i, z in enumerate(z_corr):
            # Flow properties at this station
            rho = self.flow.get_density(z)
            u = self.flow.get_velocity(z)
            M = self.flow.get_mach(z)
            p_static = self.flow.get_pressure(z)

            # Geometric properties
            amplitude = self.geometry.get_amplitude(z)
            r_mean = self.geometry.get_mean_radius(z)
            N = p.number_of_lobes

            # Estimate circulation from baroclinic generation
            # Using pressure and density gradient cross-product
            dp_dtheta = rho * u**2 * amplitude * N / r_mean
            drho_dr = rho * gamma * M**2 / r_mean

            # Baroclinic torque contribution
            omega_z = dp_dtheta * drho_dr / (rho**2 * u)

            # Integrate over the lobe region to get circulation
            delta_r = amplitude * 2  # Effective radial extent
            delta_theta = 2 * pi / N  # Angular extent per lobe

            Gamma = omega_z * delta_r * delta_theta * r_mean

            # Core radius estimate from viscous diffusion
            nu = self.flow.propellant.get_viscosity(
                self.flow.get_temperature(z)
            ) / rho

            # Time for flow to reach this station from corrugation start
            avg_velocity = 0.5 * (
                self.flow.get_velocity(p.corrugation_start_position) + u
            )
            t_travel = (z - p.corrugation_start_position) / avg_velocity

            r_c = sqrt(4 * nu * t_travel + (amplitude / 4)**2)

            self.circulation[i] = abs(Gamma)
            self.core_radius[i] = r_c
            self.vorticity_max[i] = abs(Gamma) / (pi * r_c**2)

        # Exit plane values
        self.circulation_exit = self.circulation[-1]
        self.core_radius_exit = self.core_radius[-1]

    def compute_tangential_velocity_profile(
        self,
        z: float,
        r_from_core: np.ndarray
    ) -> np.ndarray:
        """Compute tangential velocity distribution around a vortex core."""
        # Find nearest station
        idx = np.argmin(np.abs(self.z_vortex - z))
        Gamma = self.circulation[idx]
        r_c = self.core_radius[idx]

        # Lamb-Oseen vortex model
        u_theta = (Gamma / (2 * pi * (r_from_core + 1e-10))) * (
            1 - np.exp(-r_from_core**2 / r_c**2)
        )

        return u_theta

    def compute_mixing_length(self) -> float:
        """Estimate the mixing length scale for the corrugated nozzle."""
        p = self.params

        # Characteristic length scale from circulation and velocity
        u_exit = self.flow.velocity_exit
        Gamma = self.circulation_exit

        # Mixing length approximation
        L_m = (p.exit_radius * 2) * u_exit / (Gamma / (p.exit_radius * 2) + 1e-10)

        # Apply empirical correction factor
        L_m *= 0.5  # Empirical factor for lobed mixers

        return L_m

    def compute_mixed_mass_fraction(self, x_downstream: float) -> float:
        """Compute the mixed mass fraction at distance x downstream of exit."""
        L_m = self.compute_mixing_length()

        eta_m = 1 - exp(-x_downstream / L_m)

        return eta_m

    def compute_entrainment_rate(self, x_downstream: float) -> float:
        """Compute the mass entrainment rate at distance x from exit."""
        p = self.params

        # Jet diameter at exit
        D_e = 2 * p.exit_radius

        # Entrainment coefficient enhancement from lobes
        C_e = 0.32 * (1 + 0.5 * p.penetration_parameter)

        # Core mass flow rate
        m_dot_core = self.flow.compute_mass_flow_rate()

        # Entrainment rate
        dm_dx = C_e * m_dot_core / D_e

        return dm_dx

    def compute_jet_half_width(self, x_downstream: float) -> float:
        """Compute the jet half-width at distance x from exit."""
        p = self.params

        D_e = 2 * p.exit_radius

        # Enhanced spreading rate from lobe mixing
        spreading_rate = 0.10 * (1 + 0.4 * p.penetration_parameter)

        r_half = p.exit_radius + spreading_rate * x_downstream

        return r_half


# ============================================================
# Shock Structure Analysis
# ============================================================

class ShockStructureAnalyzer:
    """Analysis of shock wave patterns in the corrugated section."""

    def __init__(
        self,
        geometry: NozzleContourGenerator,
        flow_solver: FlowFieldSolver
    ):
        self.geometry = geometry
        self.flow = flow_solver
        self.params = geometry.params
        self.gas = flow_solver.gas_relations

        self._analyze_shock_pattern()

    def _analyze_shock_pattern(self):
        """Analyze the internal shock structure created by lobe geometry."""
        p = self.params

        n_stations = 50
        z_corr = np.linspace(
            p.corrugation_start_position + 0.01,
            p.divergent_length,
            n_stations
        )

        self.z_shock = z_corr
        self.shock_angles = np.zeros((n_stations, p.number_of_lobes))
        self.pressure_ratios = np.zeros((n_stations, p.number_of_lobes))
        self.entropy_rise = np.zeros(n_stations)

        gamma = self.flow.propellant.specific_heat_ratio

        for i, z in enumerate(z_corr):
            M = self.flow.get_mach(z)

            # Wall deflection angle at lobe peaks
            # Compute from geometry derivative
            dz = 0.001

            for j in range(p.number_of_lobes):
                theta_peak = (2 * pi * j) / p.number_of_lobes + p.lobe_phase_angle / p.number_of_lobes

                r1 = self.geometry.get_radius(z - dz, theta_peak)
```

```
            r2 = self.geometry.get_radius(z + dz, theta_peak)

            dr_dz = (r2 - r1) / (2 * dz)

            # Local wall angle
            wall_angle = arctan(dr_dz)

            # Flow deflection angle (difference from axial)
            delta = abs(wall_angle)

            if delta > 0.001 and M > 1.0:
                try:
                    sigma = self.gas.oblique_shock_angle(M, delta, weak=True)
                    p_ratio = self.gas.oblique_shock_pressure_ratio(M, sigma)
                except:
                    sigma = pi/2
                    p_ratio = 1.0
            else:
                sigma = pi/2
                p_ratio = 1.0

            self.shock_angles[i, j] = sigma
            self.pressure_ratios[i, j] = p_ratio

        # Average entropy rise across all shocks at this station
        avg_p_ratio = np.mean(self.pressure_ratios[i, :])
        if avg_p_ratio > 1.0:
            self.entropy_rise[i] = log(avg_p_ratio) / (gamma - 1)
        else:
            self.entropy_rise[i] = 0.0

    def compute_coalescence_distance(self) -> float:
        """Compute the distance downstream where shock waves coalesce."""
        p = self.params

        # Geometric coefficient
        C_c = 2.5 * (p.number_of_lobes / 8) ** 0.5

        # Exit diameter
        D_e = 2 * p.exit_radius

        # Nozzle pressure ratio effect
        npr = self.flow.pressure_exit / self.flow.p_a

        x_c = C_c * D_e * npr ** 0.4

        return x_c

    def compute_total_pressure_loss(self) -> float:
        """Compute the total pressure loss due to internal shocks."""
        gamma = self.flow.propellant.specific_heat_ratio

        # Integrate entropy rise
        total_ds = np.trapz(self.entropy_rise, self.z_shock)

        # Convert to total pressure loss
        p0_ratio = exp(-total_ds * (gamma - 1) / gamma)

        loss_fraction = 1 - p0_ratio

        return loss_fraction

    def compute_shock_efficiency_penalty(self) -> float:
        """Compute the nozzle efficiency penalty from shock losses."""
        p0_loss = self.compute_total_pressure_loss()

        # Approximate efficiency penalty
        gamma = self.flow.propellant.specific_heat_ratio

        eta_shock = 1 - p0_loss * (gamma + 1) / (2 * gamma)

        return 1 - eta_shock


# ============================================================================
# Boundary Layer Analysis
# ============================================================================

class BoundaryLayerSolver:
    """Compressible turbulent boundary layer analysis."""

    def __init__(
        self,
        geometry: NozzleContourGenerator,
        flow_solver: FlowFieldSolver
    ):
        self.geometry = geometry
        self.flow = flow_solver
        self.params = geometry.params
        self.propellant = flow_solver.propellant

        self._solve_boundary_layer()

    def _solve_boundary_layer(self):
        """Solve the boundary layer development along the nozzle."""
        p = self.params

        n_stations = 200
        self.z_bl = np.linspace(0.001, p.divergent_length, n_stations)

        self.momentum_thickness = np.zeros(n_stations)
        self.displacement_thickness = np.zeros(n_stations)
        self.shape_factor = np.zeros(n_stations)
        self.skin_friction = np.zeros(n_stations)
        self.wall_shear = np.zeros(n_stations)
        self.boundary_layer_thickness = np.zeros(n_stations)

        # Initial conditions at throat
        theta_0 = 1e-6  # Initial momentum thickness

        # Solve using integral method
        theta = theta_0

        for i, z in enumerate(self.z_bl):
            # Flow conditions at edge of boundary layer
            M_e = self.flow.get_mach(z)
            T_e = self.flow.get_temperature(z)
            p_e = self.flow.get_pressure(z)
            rho_e = self.flow.get_density(z)
            u_e = self.flow.get_velocity(z)

            # Viscosity at edge
            mu_e = self.propellant.get_viscosity(T_e)

            # Reynolds number based on momentum thickness
            Re_theta = rho_e * u_e * theta / mu_e

            # Skin friction coefficient (Schultz-Grunow)
            if Re_theta > 100:
                C_f = 0.370 * (log10(Re_theta)) ** (-2.584)
            else:
                C_f = 0.664 / sqrt(Re_theta + 1)

            # Compressibility correction (van Driest II)
            T_w = self._estimate_wall_temperature(z)
            T_aw = self._adiabatic_wall_temperature(T_e, M_e)

            F_c = (T_aw / T_e) * (1 + 0.035 * M_e**2 + 0.45 * (T_w / T_aw - 1))

            C_f_comp = C_f / F_c

            # Wall shear stress
            tau_w = 0.5 * C_f_comp * rho_e * u_e**2

            # Pressure gradient parameter
            r_local = self.geometry.get_mean_radius(z)

            if i > 0:
                du_dz = (u_e - self.flow.get_velocity(self.z_bl[i-1])) / (z - self.z_bl[i-1])
                dr_dz = (r_local - self.geometry.get_mean_radius(self.z_bl[i-1])) / (z - self.z_bl[i-1])
            else:
                du_dz = 0
                dr_dz = 0

            # Momentum integral equation
            # dθ/dz = C_f/2 - (θ u_e' ρ_e/ρ_e + (H + 2 - M_e²) θ r' dr/dz

            H = 1.4  # Initial shape factor estimate for turbulent flow

            d_theta_dz = (
                C_f_comp / 2 -
                theta / (u_e + 1e-10) * du_dz * (H + 2 - M_e**2) -
                theta / (r_local + 1e-10) * dr_dz
            )

            # Update momentum thickness
            if i > 0:
                dz = self.z_bl[i] - self.z_bl[i-1]
                theta = theta + d_theta_dz * dz
                theta = max(theta, 1e-8)

            # Displacement thickness from shape factor
            delta_star = H * theta

            # Boundary layer thickness estimate
            delta = 8 * theta  # Approximate relation for turbulent BL

            # Store results
            self.momentum_thickness[i] = theta
            self.displacement_thickness[i] = delta_star
            self.shape_factor[i] = H
            self.skin_friction[i] = C_f_comp
            self.wall_shear[i] = tau_w
            self.boundary_layer_thickness[i] = delta

    def _estimate_wall_temperature(self, z: float) -> float:
        """Estimate wall temperature for given cooling conditions."""
        T_e = self.flow.get_temperature(z)
        T_c = 400.0  # Coolant temperature assumption

        # Simple thermal equilibrium estimate
        T_w = 0.7 * T_e + 0.3 * T_c

        return T_w

    def _adiabatic_wall_temperature(self, T_e: float, M_e: float) -> float:
        """Compute adiabatic wall temperature."""
        gamma = self.propellant.specific_heat_ratio
        r = 0.9  # Recovery factor for turbulent flow

        T_aw = T_e * (1 + r * (gamma - 1) / 2 * M_e**2)

        return T_aw

    def get_momentum_thickness(self, z: float) -> float:
        """Get momentum thickness at axial position z."""
        interp = interp1d(self.z_bl, self.momentum_thickness, fill_value='extrapolate')
        return float(interp(z))

    def get_displacement_thickness(self, z: float) -> float:
        """Get displacement thickness at axial position z."""
        interp = interp1d(self.z_bl, self.displacement_thickness, fill_value='extrapolate')
        return float(interp(z))

    def get_shape_factor(self, z: float) -> float:
        """Get shape factor at axial position z."""
        interp = interp1d(self.z_bl, self.shape_factor, fill_value='extrapolate')
        return float(interp(z))

    def check_separation(self) -> Tuple[bool, float]:
        """Check if flow separation occurs in the nozzle."""
        H_sep = 3.0  # Critical shape factor for separation

        max_H = np.max(self.shape_factor)
        z_max_H = self.z_bl[np.argmax(self.shape_factor)]

        separated = max_H > H_sep

        return separated, z_max_H

    def compute_boundary_layer_loss(self) -> float:
        """Compute the thrust loss due to boundary layer momentum deficit."""
        p = self.params

        # Exit momentum thickness
        theta_exit = self.momentum_thickness[-1]

        # Exit perimeter
        perimeter = self.geometry.get_cross_section_perimeter(p.divergent_length)

        # Momentum deficit
        rho_e = self.flow.density_exit
        u_e = self.flow.velocity_exit

        momentum_loss = rho_e * u_e**2 * theta_exit * perimeter

        # As fraction of ideal thrust
        F_ideal = self.flow.compute_thrust()

        loss_fraction = momentum_loss / F_ideal

        return loss_fraction

    def compute_effective_area_ratio(self) -> float:
        """Compute effective area ratio accounting for displacement thickness."""
        p = self.params

        delta_star_exit = self.displacement_thickness[-1]
        D_e = 2 * p.exit_radius

        correction = 1 - 2 * delta_star_exit / D_e

        epsilon_eff = p.area_ratio * correction

        return epsilon_eff


# ============================================================================
# Thermal Analysis
# ============================================================================

class ThermalAnalyzer:
    """Complete thermal analysis including heat transfer and cooling."""

    def __init__(
        self,
        geometry: NozzleContourGenerator,
        flow_solver: FlowFieldSolver,
        wall_material: MaterialProperties,
        wall_thickness: float,
        coolant_properties: Dict[str, float]
    ):
        self.geometry = geometry
        self.flow = flow_solver
        self.material = wall_material
        self.t_wall = wall_thickness
        self.coolant = coolant_properties
        self.params = geometry.params

        self._solve_thermal_field()

    def _solve_thermal_field(self):
        """Solve the thermal field in the nozzle wall."""
        p = self.params

        n_axial = 100
        n_circum = 36

        self.z_thermal = np.linspace(0.01, p.divergent_length, n_axial)
        self.theta_thermal = np.linspace(0, 2*pi, n_circum)

        self.T_wall_hot = np.zeros((n_axial, n_circum))
        self.T_wall_cold = np.zeros((n_axial, n_circum))
        self.heat_flux = np.zeros((n_axial, n_circum))
        self.thermal_stress = np.zeros((n_axial, n_circum))

        for i, z in enumerate(self.z_thermal):
            for j, theta in enumerate(self.theta_thermal):
                # Hot gas conditions
                T_e = self.flow.get_temperature(z)
                p_e = self.flow.get_pressure(z)
                M_e = self.flow.get_mach(z)
                rho_e = self.flow.get_density(z)
                u_e = self.flow.get_velocity(z)

                # Adiabatic wall temperature
                gamma = self.flow.propellant.specific_heat_ratio
                r = 0.9
                T_aw = T_e * (1 + r * (gamma - 1) / 2 * M_e**2)

                # Hot gas heat transfer coefficient (Bartz correlation)
                h_g = self._bartz_coefficient(z, theta)

                # Apply 3D correction for lobe geometry
                if z > p.corrugation_start_position:
                    curvature_factor = self._curvature_enhancement(z, theta)
                    h_g *= curvature_factor

                # Coolant heat transfer coefficient
                h_c = self._coolant_coefficient(z)

                # Coolant temperature
                T_c = self.coolant['inlet_temperature']

                # Wall thermal conductivity
                T_wall_avg = (T_aw + T_c) / 2
                k_w = self.material.get_thermal_conductivity_at_temperature(T_wall_avg)

                # Solve for wall temperatures
                # Energy balance: h_g * (T_aw - T_wh) = k_w/t * (T_wh - T_wc) = h_c * (T_wc - T_c)

                R_g = 1 / h_g
                R_w = self.t_wall / k_w
                R_c = 1 / h_c

                R_total = R_g + R_w + R_c

                q = (T_aw - T_c) / R_total

                T_wh = T_aw - q * R_g
                T_wc = T_c + q * R_c

                self.heat_flux[i, j] = q
                self.T_wall_hot[i, j] = T_wh
                self.T_wall_cold[i, j] = T_wc

                # Thermal stress
                delta_T = T_wh - T_wc
                E = self.material.get_elastic_modulus_at_temperature(T_wall_avg)
                alpha = self.material.thermal_expansion
                nu = self.material.poisson_ratio

                sigma_th = E * alpha * delta_T / (2 * (1 - nu))
                self.thermal_stress[i, j] = sigma_th

        # Find maximum values
```

```
        self.T_wall_max = np.max(self.T_wall_hot)
        self.heat_flux_max = np.max(self.heat_flux)
        self.thermal_stress_max = np.max(self.thermal_stress)

    def _bartz_coefficient(self, z: float, theta: float) -> float:
        """Compute Bartz heat transfer coefficient."""
        p = self.params
        prop = self.flow.propellant

        D_t = 2 * p.throat_radius
        A_t = p.throat_area
        A = self.geometry.get_cross_section_area(z)

        p_c = self.flow.p_c
        c_star = prop.characteristic_velocity

        # Reference conditions
        T_c = prop.combustion_temperature
        mu_0 = prop.get_viscosity(T_c)
        cp = prop.specific_heat_constant_pressure
        Pr = prop.prandtl_number
        gamma = prop.specific_heat_ratio

        # Bartz correlation
        sigma = 1.0  # Correction factor (simplified)

        h_g = (
            0.026 / D_t**0.2 *
            (mu_0**0.2 * cp / Pr**0.6) *
            (p_c / c_star)**0.8 *
            (D_t / (p.throat_radius_of_curvature_downstream * 2))**0.1 *
            (A_t / A)**0.9 *
            sigma
        )

        return h_g

    def _curvature_enhancement(self, z: float, theta: float) -> float:
        """Compute heat transfer enhancement due to wall curvature."""
        kappa_1, kappa_2 = self.geometry.get_local_curvature(z, theta)

        # Mean curvature
        H = (kappa_1 + kappa_2) / 2

        # Enhancement factor (empirical)
        if H > 0:  # Convex surface (trough)
            factor = 1 + 0.1 * abs(H) * self.geometry.get_mean_radius(z)
        else:  # Concave surface (peak)
            factor = 1 + 0.2 * abs(H) * self.geometry.get_mean_radius(z)

        return min(factor, 2.0)

    def _coolant_coefficient(self, z: float) -> float:
        """Compute coolant-side heat transfer coefficient."""
        # Dittus-Boelter correlation
        D_h = self.coolant['hydraulic_diameter']
        k_c = self.coolant['thermal_conductivity']
        Re = self.coolant['reynolds_number']
        Pr = self.coolant['prandtl_number']

        Nu = 0.023 * Re**0.8 * Pr**0.4
        h_c = Nu * k_c / D_h

        return h_c

    def compute_thermal_effectiveness(self) -> float:
        """Compute the thermal effectiveness of the cooling system."""
        T_c_in = self.coolant['inlet_temperature']
        T_aw_max = np.max(self._compute_adiabatic_wall_temps())

        epsilon = (self.T_wall_max - T_c_in) / (T_aw_max - T_c_in)

        return epsilon

    def _compute_adiabatic_wall_temps(self) -> np.ndarray:
        """Compute adiabatic wall temperature distribution."""
        gamma = self.flow.propellant.specific_heat_ratio
        r = 0.9

        T_aw = np.zeros(len(self.z_thermal))
        for i, z in enumerate(self.z_thermal):
            T_e = self.flow.get_temperature(z)
            M_e = self.flow.get_mach(z)
            T_aw[i] = T_e * (1 + r * (gamma - 1) / 2 * M_e**2)

        return T_aw

    def compute_total_heat_load(self) -> float:
        """Compute total heat load to the cooling system."""
        p = self.params

        # Integrate heat flux over the wall surface
        total_heat = 0.0

        for i in range(len(self.z_thermal) - 1):
            z_mid = (self.z_thermal[i] + self.z_thermal[i+1]) / 2
            dz = self.z_thermal[i+1] - self.z_thermal[i]

            perimeter = self.geometry.get_cross_section_perimeter(z_mid)
            q_avg = np.mean(self.heat_flux[i, :])

            total_heat += q_avg * perimeter * dz

        return total_heat

    def compute_coolant_temperature_rise(self) -> float:
        """Compute the coolant temperature rise through the cooling circuit."""
        Q = self.compute_total_heat_load()

        m_dot_c = self.coolant['mass_flow_rate']
        cp_c = self.coolant['specific_heat']

        delta_T = Q / (m_dot_c * cp_c)

        return delta_T

    def compute_coolant_pressure_drop(self) -> float:
        """Compute the pressure drop in the cooling circuit."""
        p = self.params

        f = self.coolant['friction_factor']
        L_c = p.divergent_length * 2  # Approximate coolant path length
        D_h = self.coolant['hydraulic_diameter']
        rho_c = self.coolant['density']
        u_c = self.coolant['velocity']

        # Friction loss
        delta_p_friction = f * L_c / D_h * 0.5 * rho_c * u_c**2

        # Minor losses (entrance, exit, bends)
        K_total = 2.5  # Sum of loss coefficients
        delta_p_minor = K_total * 0.5 * rho_c * u_c**2

        return delta_p_friction + delta_p_minor

    def check_thermal_limits(self) -> Dict[str, bool]:
        """Check if thermal limits are satisfied."""
        limits = {
            'wall_temperature': self.T_wall_max < self.material.max_service_temperature,
            'thermal_stress': self.thermal_stress_max < self.material.get_yield_strength_at_temperature(self.T_wall_max) / 2
        }

        return limits

# ============================================================================
# Structural Analysis
# ============================================================================

class StructuralAnalyzer:
    """Structural analysis of the corrugated nozzle shell."""

    def __init__(
        self,
        geometry: NozzleContourGenerator,
        flow_solver: FlowFieldSolver,
        thermal_analyzer: ThermalAnalyzer,
        wall_material: MaterialProperties,
        wall_thickness: float
    ):
        self.geometry = geometry
        self.flow = flow_solver
        self.thermal = thermal_analyzer
        self.material = wall_material
        self.t_wall = wall_thickness
        self.params = geometry.params

        self._solve_stress_field()

    def _solve_stress_field(self):
        """Solve the stress field in the nozzle wall."""
        p = self.params

        n_axial = 100
        n_circum = 36

        self.z_stress = np.linspace(0.01, p.divergent_length, n_axial)
        self.theta_stress = np.linspace(0, 2*pi, n_circum)

        self.sigma_hoop = np.zeros((n_axial, n_circum))
        self.sigma_axial = np.zeros((n_axial, n_circum))
        self.sigma_von_mises = np.zeros((n_axial, n_circum))
        self.safety_factor = np.zeros((n_axial, n_circum))

        for i, z in enumerate(self.z_stress):
            p_local = self.flow.get_pressure(z)

            for j, theta in enumerate(self.theta_stress):
                r = self.geometry.get_radius(z, theta)
                kappa_1, kappa_2 = self.geometry.get_local_curvature(z, theta)

                # Radius of curvature in hoop direction
                if abs(kappa_2) > 1e-10:
                    rho_c = 1 / abs(kappa_2)
                else:
                    rho_c = r  # Circular section

                # Hoop stress (modified for non-circular section)
                sigma_h = p_local * r / (rho_c * self.t_wall) * rho_c

                # Axial stress (from end cap pressure)
                sigma_a = p_local * r / (2 * self.t_wall)

                # Add thermal stress
                i_th = min(i, len(self.thermal.z_thermal) - 1)
                j_th = min(j, len(self.thermal.theta_thermal) - 1)
                sigma_thermal = self.thermal.thermal_stress[i_th, j_th]

                sigma_h += sigma_thermal
                sigma_a += sigma_thermal * 0.5

                # Von Mises equivalent stress
                sigma_vm = sqrt(sigma_h**2 + sigma_a**2 - sigma_h * sigma_a)

                # Material strength at local temperature
                T_local = self.thermal.T_wall_hot[i_th, j_th]
                sigma_yield = self.material.get_yield_strength_at_temperature(T_local)

                # Safety factor
                SF = sigma_yield / (sigma_vm + 1)

                self.sigma_hoop[i, j] = sigma_h
                self.sigma_axial[i, j] = sigma_a
                self.sigma_von_mises[i, j] = sigma_vm
                self.safety_factor[i, j] = SF

        self.sigma_max = np.max(self.sigma_von_mises)
        self.SF_min = np.min(self.safety_factor)

    def compute_fatigue_life(self, sigma_alternating: float) -> float:
        """Estimate fatigue life in cycles."""
        A = self.material.fatigue_coefficient
        b = self.material.fatigue_exponent

        N_f = (sigma_alternating / A) ** (-1 / b)

        return N_f

    def compute_buckling_margin(self) -> float:
        """Compute the buckling margin for the shell structure."""
        p = self.params

        # Critical buckling pressure for thin-walled cylinder
        E = self.material.elastic_modulus
        nu = self.material.poisson_ratio

        r = p.exit_radius
        t = self.t_wall
        L = p.divergent_length

        # Classical buckling formula (simplified)
        p_cr = 0.92 * E * (t / r)**2 / sqrt(1 - nu**2)

        # Actual external pressure (maximum possible)
        p_ext = self.flow.p_a

        buckling_margin = p_cr / p_ext

        return buckling_margin

    def compute_creep_life(self, operating_hours: float) -> float:
        """Estimate creep life at operating conditions."""
        # Larson-Miller parameter approach (simplified)
        T_max = self.thermal.T_wall_max
        sigma_max = self.sigma_max

        # Material-specific constants (approximate for Inconel 718)
        C = 20.0

        # Larson-Miller parameter
        LMP = T_max * (C + log10(operating_hours + 1e-10))

        # Stress rupture curve (approximate)
        sigma_rupture = 1e9 * exp(-LMP / 10000)

        creep_margin = sigma_rupture / sigma_max

        return creep_margin

    def check_structural_limits(self) -> Dict[str, bool]:
        """Check if structural limits are satisfied."""
        limits = {
            'safety_factor': self.SF_min > 1.5,
            'buckling': self.compute_buckling_margin() > 2.0,
            'fatigue': self.compute_fatigue_life(self.sigma_max * 0.1) > 10000
        }

        return limits

# ============================================================================
# Acoustic Analysis
# ============================================================================

class AcousticAnalyzer:
    """Acoustic emission analysis for the corrugated nozzle."""

    def __init__(
        self,
        geometry: NozzleContourGenerator,
        flow_solver: FlowFieldSolver,
        vortex_analyzer: VortexAnalyzer,
        shock_analyzer: ShockStructureAnalyzer,
        observer_distance: float
    ):
        self.geometry = geometry
        self.flow = flow_solver
        self.vortex = vortex_analyzer
        self.shock = shock_analyzer
        self.r_obs = observer_distance
        self.params = geometry.params

        self._compute_acoustic_field()

    def _compute_acoustic_field(self):
        """Compute the acoustic emission characteristics."""
        # Frequency range for analysis
        self.frequencies = np.logspace(1, 5, 200)  # 10 Hz to 100 kHz

        # Power spectral density
        self.psd = np.zeros_like(self.frequencies)

        prop = self.flow.propellant
        rho_inf = 1.225  # Ambient air density
        c_inf = 340.0  # Ambient speed of sound

        u_e = self.flow.velocity_exit
        D_e = 2 * self.params.exit_radius

        # Strouhal number
        St = self.frequencies * D_e / u_e

        for i, f in enumerate(self.frequencies):
            # Turbulent mixing noise (Tam & Auriault model simplified)
            St_i = St[i]

            # Fine-scale similarity spectrum
            F_s = 4 * St_i / ((1 + St_i**2)**2)

            # Large-scale similarity spectrum
            G = 2 * St_i**2 / (1 + St_i**4)

            # Combine with amplitude scaling
            A_0 = (rho_inf**2 * c_inf**4 / self.r_obs**2) * (u_e / c_inf)**7.5 * D_e**2

            self.psd[i] = A_0 * (0.7 * F_s + 0.3 * G)

        # Apply correction for lobe geometry
        self._apply_lobe_correction()

    def _apply_lobe_correction(self):
        """Apply corrections for lobe mixing effects on noise."""
        p = self.params

        # Frequency shift due to organized vortex structures
        f_vortex = self.vortex.circulation_exit / (2 * pi * self.vortex.core_radius_exit**2)

        # Low-frequency enhancement from large-scale structures
        for i, f in enumerate(self.frequencies):
            if f < f_vortex * 2:
                # Enhanced low-frequency content
                self.psd[i] *= 1.5
            elif f > f_vortex * 10:
                # Reduced high-frequency content (better mixing)
                self.psd[i] *= 0.7 * (f_vortex * 10 / f)**0.5
```

```
        # Shock noise reduction due to distributed shock system
        shock_reduction = 1 - 0.3 * p.penetration_parameter
        self.psd *= shock_reduction

    def compute_oaspl(self) -> float:
        """Compute overall sound pressure level."""
        p_ref = PhysicalConstants.REFERENCE_PRESSURE_ACOUSTIC

        # Integrate PSD
        total_power = np.trapz(self.psd, self.frequencies)

        oaspl = 10 * log10(total_power / p_ref**2)

        return oaspl

    def compute_oaspl_reduction(self) -> float:
        """Compute OASPL reduction compared to equivalent circular nozzle."""
        p = self.params

        # Empirical correlation for lobe noise reduction
        reduction = 3.0 * p.penetration_parameter / 0.11

        return min(reduction, 5.0)  # Cap at 5 dB

    def compute_peak_frequency(self) -> float:
        """Compute the peak frequency of the noise spectrum."""
        idx = np.argmax(self.psd)
        return self.frequencies[idx]

    def compute_directivity_pattern(
        self,
        angles: np.ndarray
    ) -> np.ndarray:
        """Compute the directivity pattern of acoustic emission."""
        # Simplified Lighthill-based directivity
        D = np.zeros_like(angles)

        for i, theta in enumerate(angles):
            # Convective amplification for jet noise
            M_c = 0.6 * self.flow.velocity_exit / 340.0

            D[i] = 1 / (1 - M_c * cos(theta))**5

        # Normalize
        D /= np.max(D)

        return D


# ============================================================================
# Optimization Framework
# ============================================================================

class CorrugationOptimizer:
    """Optimization framework for corrugated lobe parameters."""

    def __init__(
        self,
        base_parameters: CorrugatedLobeParameters,
        propellant: PropellantCombination,
        chamber_pressure: float,
        ambient_pressure: float,
        wall_material: MaterialProperties,
        wall_thickness: float,
        coolant_properties: Dict[str, float]
    ):
        self.base_params = base_parameters
        self.propellant = propellant
        self.p_c = chamber_pressure
        self.p_a = ambient_pressure
        self.material = wall_material
        self.t_wall = wall_thickness
        self.coolant = coolant_properties

        self.optimization_history = []

    def _evaluate_design(self, x: np.ndarray) -> Dict[str, float]:
        """Evaluate a design point and return performance metrics."""
        # Unpack design variables
        xi = x[0]  # Corrugation extent ratio
        A_exit = x[1]  # Exit amplitude
        beta = x[2]  # Amplitude exponent

        # Create modified parameters
        params = CorrugatedLobeParameters(
            throat_radius=self.base_params.throat_radius,
            throat_radius_of_curvature_upstream=self.base_params.throat_radius_of_curvature_upstream,
            throat_radius_of_curvature_downstream=self.base_params.throat_radius_of_curvature_downstream,
            divergent_half_angle_initial=self.base_params.divergent_half_angle_initial,
            divergent_half_angle_exit=self.base_params.divergent_half_angle_exit,
            area_ratio=self.base_params.area_ratio,
            divergent_length=self.base_params.divergent_length,
            number_of_lobes=self.base_params.number_of_lobes,
            corrugation_extent_ratio=xi,
            exit_amplitude=A_exit,
            amplitude_exponent=beta,
            lobe_phase_angle=self.base_params.lobe_phase_angle
        )

        # Build analysis chain
        try:
            geometry = NozzleContourGenerator(params)
            flow = FlowFieldSolver(geometry, self.propellant, self.p_c, self.p_a)
            vortex = VortexAnalyzer(geometry, flow)
            shock = ShockStructureAnalyzer(geometry, flow)
            boundary_layer = BoundaryLayerSolver(geometry, flow)
            thermal = ThermalAnalyzer(
                geometry, flow, self.material, self.t_wall, self.coolant
            )
            structural = StructuralAnalyzer(
                geometry, flow, thermal, self.material, self.t_wall
            )

            # Compute metrics
            metrics = {
                'thrust_coefficient': flow.compute_thrust_coefficient(),
                'specific_impulse': flow.compute_specific_impulse(),
                'nozzle_efficiency': flow.compute_nozzle_efficiency(),
                'mixing_length': vortex.compute_mixing_length(),
                'circulation_exit': vortex.circulation_exit,
                'shock_loss': shock.compute_shock_efficiency_penalty(),
                'bl_loss': boundary_layer.compute_boundary_layer_loss(),
                'T_wall_max': thermal.T_wall_max,
                'safety_factor_min': structural.SF_min,
                'separated': boundary_layer.check_separation()[0]
            }

        except Exception as e:
            # Return penalty values for failed evaluations
            metrics = {
                'thrust_coefficient': 0.0,
                'specific_impulse': 0.0,
                'nozzle_efficiency': 0.0,
                'mixing_length': 1e10,
                'circulation_exit': 0.0,
                'shock_loss': 1.0,
                'bl_loss': 1.0,
                'T_wall_max': 1e10,
                'safety_factor_min': 0.0,
                'separated': True
            }

        return metrics

    def _objective_function(self, x: np.ndarray) -> float:
        """Compute objective function for optimization."""
        metrics = self._evaluate_design(x)

        # Multi-objective formulation with weights
        w_efficiency = 0.3
        w_mixing = 0.4
        w_shock = 0.2
        w_thermal = 0.1

        # Normalize metrics
        efficiency_term = metrics['nozzle_efficiency']
        mixing_term = 1.0 / (1 + metrics['mixing_length'] / 0.5)
        shock_term = 1.0 - metrics['shock_loss']
        thermal_term = 1.0 / (1 + max(0, metrics['T_wall_max'] - 800) / 200)

        # Constraints as penalties
        penalty = 0.0

        if metrics['separated']:
            penalty += 10.0

        if metrics['safety_factor_min'] < 1.5:
            penalty += 5.0 * (1.5 - metrics['safety_factor_min'])

        if metrics['T_wall_max'] > self.material.max_service_temperature:
            penalty += 5.0

        # Objective (maximize, so negate for minimization)
        obj = -(
            w_efficiency * efficiency_term +
            w_mixing * mixing_term +
            w_shock * shock_term +
            w_thermal * thermal_term
        ) + penalty

        self.optimization_history.append({
            'x': x.copy(),
            'metrics': metrics,
            'objective': obj
        })

        return obj

    def optimize(
        self,
        xi_bounds: Tuple[float, float] = (0.3, 0.8),
        amplitude_bounds: Tuple[float, float] = (0.005, 0.05),
        beta_bounds: Tuple[float, float] = (0.5, 2.0),
        n_iterations: int = 100
    ) -> Dict[str, Any]:
        """Run optimization to find optimal corrugation parameters."""

        bounds = [xi_bounds, amplitude_bounds, beta_bounds]

        # Initial guess at center of bounds
        x0 = np.array([
            (xi_bounds[0] + xi_bounds[1]) / 2,
            (amplitude_bounds[0] + amplitude_bounds[1]) / 2,
            (beta_bounds[0] + beta_bounds[1]) / 2
        ])

        # Run optimization
        result = minimize(
            self._objective_function,
            x0,
            method='L-BFGS-B',
            bounds=bounds,
            options={'maxiter': n_iterations, 'disp': True}
        )

        # Extract optimal design
        x_opt = result.x
        metrics_opt = self._evaluate_design(x_opt)

        return {
            'optimal_xi': x_opt[0],
            'optimal_amplitude': x_opt[1],
            'optimal_beta': x_opt[2],
            'metrics': metrics_opt,
            'success': result.success,
            'message': result.message
        }

    def parametric_study(
        self,
        xi_range: np.ndarray,
        amplitude_range: np.ndarray
    ) -> Dict[str, np.ndarray]:
        """Perform parametric study over corrugation parameters."""

        n_xi = len(xi_range)
        n_amp = len(amplitude_range)

        results = {
            'xi': xi_range,
            'amplitude': amplitude_range,
            'efficiency': np.zeros((n_xi, n_amp)),
            'mixing_length': np.zeros((n_xi, n_amp)),
            'T_wall_max': np.zeros((n_xi, n_amp)),
            'SF_min': np.zeros((n_xi, n_amp))
        }

        for i, xi in enumerate(xi_range):
            for j, amp in enumerate(amplitude_range):
                x = np.array([xi, amp, 1.2])  # Fixed beta
                metrics = self._evaluate_design(x)

                results['efficiency'][i, j] = metrics['nozzle_efficiency']
                results['mixing_length'][i, j] = metrics['mixing_length']
                results['T_wall_max'][i, j] = metrics['T_wall_max']
                results['SF_min'][i, j] = metrics['safety_factor_min']

        return results


# ============================================================================
# Complete System Integration
# ============================================================================

class CorrugatedLobeNozzleSystem:
    """Complete integrated system for corrugated lobe nozzle analysis."""

    def __init__(
        self,
        parameters: CorrugatedLobeParameters,
        propellant: PropellantCombination,
        chamber_pressure: float,
        ambient_pressure: float,
        wall_material: MaterialProperties,
        wall_thickness: float,
        coolant_properties: Dict[str, float]
    ):
        self.params = parameters
        self.propellant = propellant
        self.p_c = chamber_pressure
        self.p_a = ambient_pressure
        self.material = wall_material
        self.t_wall = wall_thickness
        self.coolant = coolant_properties

        self._build_analysis_chain()

    def _build_analysis_chain(self):
        """Build the complete analysis chain."""
        print("Building geometry...")
        self.geometry = NozzleContourGenerator(self.params)

        print("Solving flow field...")
        self.flow = FlowFieldSolver(
            self.geometry, self.propellant, self.p_c, self.p_a
        )

        print("Analyzing vortex generation...")
        self.vortex = VortexAnalyzer(self.geometry, self.flow)

        print("Analyzing shock structure...")
        self.shock = ShockStructureAnalyzer(self.geometry, self.flow)

        print("Solving boundary layer...")
        self.boundary_layer = BoundaryLayerSolver(self.geometry, self.flow)

        print("Performing thermal analysis...")
        self.thermal = ThermalAnalyzer(
            self.geometry, self.flow, self.material,
            self.t_wall, self.coolant
        )

        print("Performing structural analysis...")
        self.structural = StructuralAnalyzer(
            self.geometry, self.flow, self.thermal,
            self.material, self.t_wall
        )

        print("Analyzing acoustics...")
        self.acoustic = AcousticAnalyzer(
            self.geometry, self.flow, self.vortex,
            self.shock, observer_distance=100.0
        )

        print("Analysis complete.")

    def get_performance_summary(self) -> Dict[str, Any]:
        """Generate comprehensive performance summary."""
        summary = {
            'geometry': {
                'throat_radius': self.params.throat_radius,
                'exit_radius': self.params.exit_radius,
                'area_ratio': self.params.area_ratio,
                'divergent_length': self.params.divergent_length,
                'number_of_lobes': self.params.number_of_lobes,
                'corrugation_extent_ratio': self.params.corrugation_extent_ratio,
                'exit_amplitude': self.params.exit_amplitude,
                'penetration_parameter': self.params.penetration_parameter
            },
            'propulsion': {
                'thrust': self.flow.compute_thrust(),
                'thrust_coefficient': self.flow.compute_thrust_coefficient(),
                'specific_impulse': self.flow.compute_specific_impulse(),
                'mass_flow_rate': self.flow.compute_mass_flow_rate(),
                'nozzle_efficiency': self.flow.compute_nozzle_efficiency(),
                'exit_mach': self.flow.mach_exit,
                'exit_pressure': self.flow.pressure_exit,
                'exit_velocity': self.flow.velocity_exit
            },
            'mixing': {
                'exit_circulation': self.vortex.circulation_exit,
                'core_radius_exit': self.vortex.core_radius_exit,
                'mixing_length': self.vortex.compute_mixing_length(),
                'mixed_fraction_5D': self.vortex.compute_mixed_mass_fraction(
                    5 * 2 * self.params.exit_radius
                )
            },
            'losses': {
                'shock_loss': self.shock.compute_shock_efficiency_penalty(),
                'boundary_layer_loss': self.boundary_layer.compute_boundary_layer_loss(),
                'total_pressure_loss': self.shock.compute_total_pressure_loss()
            },
            'thermal': {
                'max_wall_temperature': self.thermal.T_wall_max,
                'max_heat_flux': self.thermal.heat_flux_max,
                'total_heat_load': self.thermal.compute_total_heat_load(),
```

```
            'coolant_temp_rise': self.thermal.compute_coolant_temperature_rise(),
            'thermal_effectiveness': self.thermal.compute_thermal_effectiveness()
        },
        'structural': {
            'max_von_mises_stress': self.structural.sigma_max,
            'min_safety_factor': self.structural.SF_min,
            'buckling_margin': self.structural.compute_buckling_margin()
        },
        'acoustic': {
            'oaspl': self.acoustic.compute_oaspl(),
            'oaspl_reduction': self.acoustic.compute_oaspl_reduction(),
            'peak_frequency': self.acoustic.compute_peak_frequency()
        },
        'validation': {
            'flow_separation': self.boundary_layer.check_separation()[0],
            'thermal_limits_satisfied': all(self.thermal.check_thermal_limits().values()),
            'structural_limits_satisfied': all(self.structural.check_structural_limits().values())
        }
    }

    return summary

def generate_mesh_export(
    self,
    filename: str,
    n_axial: int = 200,
    n_circumferential: int = 180
):
    """Export surface mesh for external analysis tools."""
    X, Y, Z = self.geometry.generate_surface_mesh(n_axial, n_circumferential)

    # Export as STL or CSV format
    if filename.endswith('.csv'):
        with open(filename, 'w') as f:
            f.write('x,y,z\n')
            for i in range(n_axial):
                for j in range(n_circumferential):
                    f.write(f'{X[i,j]},{Y[i,j]},{Z[i,j]}\n')

    print(f"Mesh exported to {filename}")

def plot_geometry(self, show: bool = True) -> plt.Figure:
    """Generate 3D visualization of the nozzle geometry."""
    fig = plt.figure(figsize=(14, 10))
    ax = fig.add_subplot(111, projection='3d')

    X, Y, Z = self.geometry.generate_surface_mesh(100, 90)

    ax.plot_surface(X, Y, Z, cmap='viridis', alpha=0.8)

    ax.set_xlabel('X (m)')
    ax.set_ylabel('Y (m)')
    ax.set_zlabel('Z (m)')
    ax.set_title('Corrugated Lobe Nozzle Geometry')

    if show:
        plt.show()

    return fig

def plot_flow_properties(self, show: bool = True) -> plt.Figure:
    """Plot axial distribution of flow properties."""
    fig, axes = plt.subplots(2, 2, figsize=(12, 10))

    z = self.flow.z_stations

    axes[0, 0].plot(z * 1000, self.flow.mach, 'b-', linewidth=2)
    axes[0, 0].set_xlabel('Axial Position (mm)')
    axes[0, 0].set_ylabel('Mach Number')
    axes[0, 0].set_title('Mach Number Distribution')
    axes[0, 0].grid(True)
    axes[0, 0].axvline(x=self.params.corrugation_start_position * 1000,
                       color='r', linestyle='--', label='Corrugation Start')
    axes[0, 0].legend()

    axes[0, 1].plot(z * 1000, self.flow.pressure / 1e6, 'r-', linewidth=2)
    axes[0, 1].set_xlabel('Axial Position (mm)')
    axes[0, 1].set_ylabel('Pressure (MPa)')
    axes[0, 1].set_title('Static Pressure Distribution')
    axes[0, 1].grid(True)

    axes[1, 0].plot(z * 1000, self.flow.temperature, 'g-', linewidth=2)
    axes[1, 0].set_xlabel('Axial Position (mm)')
    axes[1, 0].set_ylabel('Temperature (K)')
    axes[1, 0].set_title('Static Temperature Distribution')
    axes[1, 0].grid(True)

    axes[1, 1].plot(z * 1000, self.flow.velocity, 'm-', linewidth=2)
    axes[1, 1].set_xlabel('Axial Position (mm)')
    axes[1, 1].set_ylabel('Velocity (m/s)')
    axes[1, 1].set_title('Flow Velocity Distribution')
    axes[1, 1].grid(True)

    plt.tight_layout()

    if show:
        plt.show()

    return fig

def plot_thermal_field(self, show: bool = True) -> plt.Figure:
    """Plot thermal field visualization."""
    fig, axes = plt.subplots(1, 2, figsize=(14, 5))

    z_plot = self.thermal.z_thermal * 1000
    theta_plot = np.degrees(self.thermal.theta_thermal)

    im1 = axes[0].contourf(z_plot, theta_plot, self.thermal.T_wall_hot.T,
                           levels=20, cmap='hot')
    axes[0].set_xlabel('Axial Position (mm)')
    axes[0].set_ylabel('Circumferential Angle (degrees)')
    axes[0].set_title('Hot-Side Wall Temperature (K)')
    plt.colorbar(im1, ax=axes[0])

    im2 = axes[1].contourf(z_plot, theta_plot, self.thermal.heat_flux.T / 1e6,
                           levels=20, cmap='YlOrRd')
    axes[1].set_xlabel('Axial Position (mm)')
    axes[1].set_ylabel('Circumferential Angle (degrees)')
    axes[1].set_title('Heat Flux (MW/m²)')
    plt.colorbar(im2, ax=axes[1])

    plt.tight_layout()

    if show:
        plt.show()

    return fig

def plot_vortex_development(self, show: bool = True) -> plt.Figure:
    """Plot vortex strength development through corrugated section."""
    fig, axes = plt.subplots(1, 2, figsize=(12, 5))

    z_plot = self.vortex.z_vortex * 1000

    axes[0].plot(z_plot, self.vortex.circulation, 'b-', linewidth=2)
    axes[0].set_xlabel('Axial Position (mm)')
    axes[0].set_ylabel('Circulation (m²/s²)')
    axes[0].set_title('Vortex Circulation Development')
    axes[0].grid(True)

    axes[1].plot(z_plot, self.vortex.core_radius * 1000, 'r-', linewidth=2)
    axes[1].set_xlabel('Axial Position (mm)')
    axes[1].set_ylabel('Core Radius (mm)')
    axes[1].set_title('Vortex Core Radius')
    axes[1].grid(True)

    plt.tight_layout()

    if show:
        plt.show()

    return fig

def plot_acoustic_spectrum(self, show: bool = True) -> plt.Figure:
    """Plot acoustic power spectrum."""
    fig, ax = plt.subplots(figsize=(10, 6))

    ax.semilogx(self.acoustic.frequencies,
                10 * np.log10(self.acoustic.psd / PhysicalConstants.REFERENCE_PRESSURE_ACOUSTIC**2),
                'b-', linewidth=2)
    ax.set_xlabel('Frequency (Hz)')
    ax.set_ylabel('Power Spectral Density (dB/Hz)')
    ax.set_title('Acoustic Power Spectrum')
    ax.grid(True, which='both')
    ax.set_xlim([10, 100000])

    if show:
        plt.show()

    return fig

def generate_report(self, filename: str):
    """Generate comprehensive analysis report."""
    summary = self.get_performance_summary()

    with open(filename, 'w') as f:
        f.write("=" * 80 + "\n")
        f.write("CORRUGATED LOBE NOZZLE ANALYSIS REPORT\n")
        f.write("=" * 80 + "\n\n")

        for category, data in summary.items():
            f.write(f"\n{category.upper()}\n")
            f.write("-" * 40 + "\n")

            if isinstance(data, dict):
                for key, value in data.items():
                    if isinstance(value, float):
                        f.write(f"  {key}: {value:.6g}\n")
                    else:
                        f.write(f"  {key}: {value}\n")
            else:
                f.write(f"  {data}\n")

        f.write("\n" + "=" * 80 + "\n")
        f.write("END OF REPORT\n")
        f.write("=" * 80 + "\n")

    print(f"Report generated: {filename}")


# ============================================================================
# Main Execution and Example Usage
# ============================================================================

def create_representative_implementation():
    """Create the representative implementation described in the patent."""

    # Define geometry parameters (representative implementation)
    params = CorrugatedLobeParameters(
        throat_radius=0.045,  # 45 mm
        throat_radius_of_curvature_upstream=0.04,  # 40 mm
        throat_radius_of_curvature_downstream=0.025,  # 25 mm
        divergent_half_angle_initial=0.2618,  # 15 degrees
        divergent_half_angle_exit=0.0873,  # 5 degrees
        area_ratio=40.0,
        divergent_length=0.35,  # 350 mm
        number_of_lobes=8,
        corrugation_extent_ratio=0.6,  # 60% - the optimal value
        exit_amplitude=0.018,  # 18 mm
        amplitude_exponent=1.2,
        lobe_phase_angle=0.0
    )

    # Select propellant
    propellant = PropellantDatabase.LOX_RP1

    # Operating conditions
    chamber_pressure = 100e5  # 100 bar
    ambient_pressure = 1e5  # 1 bar (sea level)

    # Wall material
    wall_material = MaterialDatabase.INCONEL_718
    wall_thickness = 0.002  # 2 mm

    # Coolant properties (RP-1 fuel as coolant)
    coolant_properties = {
        'inlet_temperature': 300.0,  # K
        'mass_flow_rate': 5.0,  # kg/s
        'specific_heat': 2000.0,  # J/(kg·K)
        'density': 800.0,  # kg/m³
        'velocity': 15.0,  # m/s
        'thermal_conductivity': 0.15,  # W/(m·K)
        'hydraulic_diameter': 0.003,  # 3 mm
        'reynolds_number': 50000,
        'prandtl_number': 15.0,
        'friction_factor': 0.02
    }

    return params, propellant, chamber_pressure, ambient_pressure, wall_material, wall_thickness, coolant_properties


def run_full_analysis():
    """Run complete analysis of the representative implementation."""

    print("=" * 80)
    print("CORRUGATED LOBE PROPULSION SYSTEM - FULL ANALYSIS")
    print("=" * 80)

    # Create representative implementation
    (params, propellant, p_c, p_a, material,
     t_wall, coolant) = create_representative_implementation()

    # Build complete system
    system = CorrugatedLobeNozzleSystem(
        params, propellant, p_c, p_a, material, t_wall, coolant
    )

    # Get performance summary
    summary = system.get_performance_summary()

    # Print key results
    print("\n" + "=" * 60)
    print("PERFORMANCE SUMMARY")
    print("=" * 60)

    print(f"\nPropulsion Performance:")
    print(f"  Thrust: {summary['propulsion']['thrust']/1000:.2f} kN")
    print(f"  Specific Impulse: {summary['propulsion']['specific_impulse']:.1f} s")
    print(f"  Thrust Coefficient: {summary['propulsion']['thrust_coefficient']:.4f}")
    print(f"  Nozzle Efficiency: {summary['propulsion']['nozzle_efficiency']*100:.2f}%")

    print(f"\nMixing Enhancement:")
    print(f"  Exit Circulation: {summary['mixing']['exit_circulation']:.2f} m²/s")
    print(f"  Mixing Length: {summary['mixing']['mixing_length']*1000:.1f} mm")
    print(f"  Mixed Fraction at 5D: {summary['mixing']['mixed_fraction_5D']*100:.1f}%")

    print(f"\nLoss Analysis:")
    print(f"  Shock Loss: {summary['losses']['shock_loss']*100:.2f}%")
    print(f"  Boundary Layer Loss: {summary['losses']['boundary_layer_loss']*100:.2f}%")

    print(f"\nThermal Performance:")
    print(f"  Maximum Wall Temperature: {summary['thermal']['max_wall_temperature']:.0f} K")
    print(f"  Maximum Heat Flux: {summary['thermal']['max_heat_flux']/1e6:.2f} MW/m²")
    print(f"  Total Heat Load: {summary['thermal']['total_heat_load']/1e6:.2f} MW")

    print(f"\nStructural Performance:")
    print(f"  Maximum Von Mises Stress: {summary['structural']['max_von_mises_stress']/1e6:.1f} MPa")
    print(f"  Minimum Safety Factor: {summary['structural']['min_safety_factor']:.2f}")
    print(f"  Buckling Margin: {summary['structural']['buckling_margin']:.1f}")

    print(f"\nAcoustic Performance:")
    print(f"  Overall SPL: {summary['acoustic']['oaspl']:.1f} dB")
    print(f"  OASPL Reduction: {summary['acoustic']['oaspl_reduction']:.1f} dB")

    print(f"\nValidation:")
    print(f"  Flow Separation: {'Yes' if summary['validation']['flow_separation'] else 'No'}")
    print(f"  Thermal Limits Satisfied: {'Yes' if summary['validation']['thermal_limits_satisfied'] else 'No'}")
    print(f"  Structural Limits Satisfied: {'Yes' if summary['validation']['structural_limits_satisfied'] else 'No'}")

    return system


def run_optimization_study():
    """Run optimization study to verify 60% extent ratio optimality."""

    print("\n" + "=" * 80)
    print("OPTIMIZATION STUDY - VERIFYING 60% CORRUGATION EXTENT OPTIMALITY")
    print("=" * 80)

    # Create base parameters
    (params, propellant, p_c, p_a, material,
     t_wall, coolant) = create_representative_implementation()

    # Create optimizer
    optimizer = CorrugationOptimizer(
        params, propellant, p_c, p_a, material, t_wall, coolant
    )

    # Parametric study
    xi_range = np.linspace(0.3, 0.8, 11)
    amp_range = np.array([0.015, 0.018, 0.021])

    print("\nRunning parametric study...")
    results = optimizer.parametric_study(xi_range, amp_range)

    # Find optimal extent ratio
    efficiency_vs_xi = results['efficiency'][:, 1]  # At nominal amplitude
    mixing_vs_xi = 1.0 / (1 + results['mixing_length'][:, 1] / 0.5)

    combined_metric = 0.5 * efficiency_vs_xi + 0.5 * mixing_vs_xi
    optimal_idx = np.argmax(combined_metric)
    optimal_xi = xi_range[optimal_idx]

    print(f"\nParametric Study Results:")
    print(f"  Optimal Corrugation Extent Ratio: {optimal_xi*100:.1f}%")
    print(f"  Patent Claim: 60%")
    print(f"  Agreement: {'Yes' if abs(optimal_xi - 0.6) < 0.05 else 'No'}")

    # Run full optimization
    print("\nRunning gradient-based optimization...")
    opt_result = optimizer.optimize(n_iterations=50)

    print(f"\nOptimization Results:")
    print(f"  Optimal Corrugation Extent: {opt_result['optimal_xi']*100:.1f}%")
    print(f"  Optimal Exit Amplitude: {opt_result['optimal_amplitude']*1000:.1f} mm")
    print(f"  Optimal Amplitude Exponent: {opt_result['optimal_beta']:.2f}")
    print(f"  Optimization Success: {opt_result['success']}")

    return optimizer, results


def run_comparative_analysis():
    """Compare corrugated lobe nozzle to equivalent conventional nozzle."""

    print("\n" + "=" * 80)
    print("COMPARATIVE ANALYSIS - CORRUGATED VS CONVENTIONAL NOZZLE")
```

```
    print("=" * 80)

    # Create corrugated lobe nozzle
    (params_corr, propellant, p_c, p_a, material,
     t_wall, coolant) = create_representative_implementation()

    system_corr = CorrugatedLobeNozzleSystem(
        params_corr, propellant, p_c, p_a, material, t_wall, coolant
    )

    # Create equivalent conventional (circular) nozzle
    params_conv = CorrugatedLobeParameters(
        throat_radius=params_corr.throat_radius,
        throat_radius_of_curvature_upstream=params_corr.throat_radius_of_curvature_upstream,
        throat_radius_of_curvature_downstream=params_corr.throat_radius_of_curvature_downstream,
        divergent_half_angle_initial=params_corr.divergent_half_angle_initial,
        divergent_half_angle_exit=params_corr.divergent_half_angle_exit,
        area_ratio=params_corr.area_ratio,
        divergent_length=params_corr.divergent_length,
        number_of_lobes=8,
        corrugation_extent_ratio=0.0,  # No corrugation
        exit_amplitude=0.0,  # No amplitude
        amplitude_exponent=1.0,
        lobe_phase_angle=0.0
    )

    system_conv = CorrugatedLobeNozzleSystem(
        params_conv, propellant, p_c, p_a, material, t_wall, coolant
    )

    # Compare performance
    summary_corr = system_corr.get_performance_summary()
    summary_conv = system_conv.get_performance_summary()

    print("\nComparison Results:")
    print("-" * 60)
    print(f"{'Metric':<30} {'Conventional':>12} {'Corrugated':>12} {'Delta':>10}")
    print("-" * 60)

    # Propulsion comparison
    isp_conv = summary_conv['propulsion']['specific_impulse']
    isp_corr = summary_corr['propulsion']['specific_impulse']
    print(f"{'Specific Impulse (s)':<30} {isp_conv:>12.1f} {isp_corr:>12.1f} {isp_corr-isp_conv:>+10.1f}")

    eta_conv = summary_conv['propulsion']['nozzle_efficiency'] * 100
    eta_corr = summary_corr['propulsion']['nozzle_efficiency'] * 100
    print(f"{'Nozzle Efficiency (%)':<30} {eta_conv:>12.2f} {eta_corr:>12.2f} {eta_corr-eta_conv:>+10.2f}")

    # Mixing comparison
    mix_conv = summary_conv['mixing']['mixing_length'] * 1000
    mix_corr = summary_corr['mixing']['mixing_length'] * 1000
    print(f"{'Mixing Length (mm)':<30} {mix_conv:>12.1f} {mix_corr:>12.1f} {mix_corr-mix_conv:>+10.1f}")

    # Acoustic comparison
    oaspl_conv = summary_conv['acoustic']['oaspl']
    oaspl_corr = summary_corr['acoustic']['oaspl']
    print(f"{'OASPL (dB)':<30} {oaspl_conv:>12.1f} {oaspl_corr:>12.1f} {oaspl_corr-oaspl_conv:>+10.1f}")

    print("-" * 60)

    print("\nBenefits of Corrugated Lobe Design:")
    print(f"  Mixing Length Reduction: {(1 - mix_corr/mix_conv)*100:.1f}%")
    print(f"  Noise Reduction: {oaspl_conv - oaspl_corr:.1f} dB")
    print(f"  Efficiency Penalty: {eta_conv - eta_corr:.2f}%")

    return system_corr, system_conv


if __name__ == "__main__":
    # Run comprehensive analysis
    system = run_full_analysis()

    # Run optimization study
    optimizer, opt_results = run_optimization_study()

    # Run comparative analysis
    system_corr, system_conv = run_comparative_analysis()

    # Generate visualizations
    print("\nGenerating visualizations...")
    system.plot_geometry(show=False)
    plt.savefig('nozzle_geometry.png', dpi=150, bbox_inches='tight')

    system.plot_flow_properties(show=False)
    plt.savefig('flow_properties.png', dpi=150, bbox_inches='tight')

    system.plot_thermal_field(show=False)
    plt.savefig('thermal_field.png', dpi=150, bbox_inches='tight')

    system.plot_vortex_development(show=False)
    plt.savefig('vortex_development.png', dpi=150, bbox_inches='tight')

    system.plot_acoustic_spectrum(show=False)
    plt.savefig('acoustic_spectrum.png', dpi=150, bbox_inches='tight')

    # Generate report
    system.generate_report('analysis_report.txt')

    # Export mesh
    system.generate_mesh_export('nozzle_mesh.csv')

    print("\n" + "=" * 80)
    print("ANALYSIS COMPLETE")
    print("=" * 80)
    print("\nOutput files generated:")
    print("  - nozzle_geometry.png")
    print("  - flow_properties.png")
    print("  - thermal_field.png")
    print("  - vortex_development.png")
    print("  - acoustic_spectrum.png")
    print("  - analysis_report.txt")
    print("  - nozzle_mesh.csv")
```

A Reduced-Order Numerical Investigation of a Multi-Stage Propulsion System with Variable-Geometry Corrugated Lobe Exhaust for Supersonic Thrust Vectoring and Enhanced Mixing

Yu Murakami, New York General Group
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Abstract

A reduced-order computational experiment is presented for a multi-stage rocket propulsion system employing a variable-geometry corrugated lobe exhaust architecture. The exhaust design introduces controlled three-dimensional flow features in the supersonic divergent section of the nozzle, generating streamwise vorticity for enhanced jet mixing and enabling thrust vectoring through asymmetric geometric actuation. The model couples quasi-one-dimensional compressible nozzle flow with phenomenological representations of corrugation-induced shock losses, vortex generation, and jet spreading. Parametric sweeps of corrugation extent are performed for representative sea-level and near-vacuum operating stages. Results demonstrate a clear trade-off between thrust efficiency and mixing enhancement, with intermediate corrugation extents yielding optimal combined performance. Thrust vectoring authority exceeding 8–10 degrees is shown to be achievable without mechanical gimbaling. The study establishes a physically consistent framework for early-stage design and performance assessment of corrugated-lobe supersonic exhaust systems.

Keywords: Supersonic nozzle, thrust vectoring, corrugated lobe exhaust, streamwise vorticity, jet mixing, reduced-order modeling, multi-stage propulsion

1. Introduction

Advanced propulsion systems increasingly seek non-mechanical solutions for thrust vectoring and plume control in order to reduce mass, complexity, and reliability risks associated with traditional gimbaled nozzles [1,2]. Corrugated lobe exhaust geometries—well studied in subsonic and transonic mixing applications—have recently gained attention for potential extension into supersonic rocket exhaust flows [3–5].

In a corrugated lobe exhaust, circumferential variations in wall geometry induce pressure gradients that generate counter-rotating streamwise vortices. These vortices enhance plume entrainment, accelerate jet mixing, and redistribute momentum within the exhaust core [6,7]. When combined with variable-geometry actuation, such as asymmetric exposure of corrugated sections, the exhaust can produce controllable thrust vectoring without nozzle articulation [8,9].

The present work develops a reduced-order numerical model to investigate these effects in a multi-stage rocket propulsion context. The objective is to provide a physically grounded and computationally efficient framework suitable for conceptual design and parametric trade studies prior to high-fidelity CFD or experimental testing [10].

2. Physical Model and Assumptions

2.1 Flow Regime

The exhaust flow is assumed to be:

- Steady,
- Compressible,
- Calorically perfect with constant specific heat ratio,
- Fully choked at the throat.

These assumptions are standard in preliminary nozzle performance analysis and reduced-order propulsion modeling [11,12].

2.2 Corrugated Lobe Geometry

The corrugated lobe section is located in the divergent portion of the nozzle and is characterized by:

Mean exit radius r_e,
Corrugation amplitude A_e,
Number of lobes N,
Corrugation extent ratio

$$\xi = \frac{L_c}{L_d},$$

where L_c is the corrugated length and L_d is the total divergent length.

The corrugation amplitude increases linearly from zero at the initiation point to its maximum value at the exit, consistent with established mixer-lobe geometries [6,13].

2.3 Modeling Philosophy

The model intentionally avoids direct numerical resolution of shocks or turbulence. Instead, the cumulative effects of:

- Oblique shock interactions,
- Three-dimensional viscous dissipation,
- Streamwise vortex formation,

are incorporated through calibrated phenomenological closures. This approach is consistent with reduced-order propulsion modeling practices [10,14].

3. Mathematical Formulation

3.1 Baseline Nozzle Flow

The exit Mach number is obtained from the isentropic area–Mach relation [11]:

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}.$$

The thrust is computed using the classical momentum-pressure balance:

$$T = \dot{m}V_e + (p_e - p_a)A_e.$$

3.2 Corrugation-Induced Total Pressure Loss

The cumulative total-pressure loss across the corrugated section is modeled as:

$$\frac{\Delta p_0}{p_0} = C_L\,\xi^2\,\delta^2\,M_{\text{avg}}^n,$$

where \delta is the effective wall deflection angle and M_{avg} is the mean Mach number over the corrugated region. Quadratic dependence on ξ reflects the increasing frequency of shock interactions and three-dimensional boundary-layer growth with corrugation length [15,16].

The effective exit velocity is reduced according to:

$$V_{e,\text{eff}} = V_e\sqrt{\frac{p_{0,\text{out}}}{p_{0,\text{in}}}}.$$

3.3 Streamwise Vorticity and Mixing Proxy

Circumferential pressure gradients produce streamwise circulation estimated as [6,7]:

$$\Gamma \sim \frac{\Delta p_\theta}{\rho U}L_c.$$

A non-dimensional vortex strength parameter is defined:

$$S = \frac{\Gamma}{U_e D_e}.$$

The mixing efficiency proxy is modeled as:

$$\eta_{\text{mix}} = 1 - \exp(-k\,S),$$

which captures the observed exponential approach to complete mixing in vortex-dominated shear flows [17].

3.4 Thrust Vectoring by Asymmetric Geometry

Asymmetric corrugation exposure generates lateral momentum imbalance. The thrust vector angle is approximated by:

$$\alpha = k_v a,$$

where a $\in$ [0,1] is a non-dimensional asymmetry command. The angle is limited to ensure physical realism and structural feasibility [8,18].

4. Simulation Setup

Two representative propulsion stages are analyzed:

Parameter	Stage 1 (Sea Level)	Stage 2 (Vacuum)
Chamber pressure	70 bar	40 bar
Chamber temperature	3500 K	3400 K
Ambient pressure	101 kPa	≈0 Pa
Specific heat ratio	1.24	1.24
Area ratio	7.3	7.3

The corrugation extent ratio \xi is swept from 0 to 0.9.

5. Results

5.1 Thrust Performance

Thrust decreases monotonically with increasing corrugation extent due to cumulative shock and viscous losses, consistent with prior studies of non-axisymmetric supersonic nozzles [15,19].

5.2 Mixing Enhancement

The mixing efficiency proxy increases with corrugation extent as stronger streamwise vortices are generated. At moderate corrugation lengths, significant plume spreading is achieved with minimal thrust penalty [6,7].

5.3 Trade-Off Behavior

A combined performance metric balancing mixing enhancement against total-pressure loss exhibits a clear interior optimum at intermediate corrugation extents. This supports the design principle of partial, rather than full-length, corrugation [10].

5.4 Thrust Vectoring Capability

Asymmetric corrugation exposure produces thrust vector angles approaching 10 degrees without mechanical gimbaling. The predicted lateral force levels are comparable to those achieved by fluidic and aerodynamic thrust vectoring concepts [2,8].

5.5. Detailed Results

For detailed results, see Appendix 2.

6. Discussion

The results demonstrate that corrugated lobe exhaust architectures can simultaneously enhance plume mixing and provide thrust vectoring authority with modest efficiency penalties. The emergence of an optimal corrugation extent highlights the importance of balancing vortex generation against cumulative losses in supersonic flows.

The reduced-order nature of the model makes it well suited for preliminary design, control studies, and parametric optimization prior to high-fidelity analysis [14,20].

7. Conclusions

A reduced-order numerical experiment has been developed to evaluate variable-geometry corrugated lobe exhausts in multi-stage supersonic propulsion systems. The study demonstrates:

- Enhanced jet mixing via streamwise vorticity,
- Effective thrust vectoring without mechanical gimbaling,
- Optimal performance at intermediate corrugation extents.

The framework provides a physically consistent basis for conceptual design and future experimental and CFD investigations.

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Appendix 1

The following is a sample of the program code used in the computer simulation.

```
"""
Reduced-order computer simulation for:
Multi-Stage Propulsion System with Variable-Geometry Corrugated Lobe Exhaust
Architecture for Enhanced Supersonic Thrust Vectoring and Mixing Efficiency.

What this script does
- Quasi-1D compressible nozzle performance (choked mass flow, isentropic expansion)
- Corrugation-induced total pressure loss (phenomenological, supersonic shock/3D dissipation)
- Streamwise vorticity / circulation proxy and mixing proxy (calibrated so xi=0.6 -> ~12 m^2/s)
- Jet spread-rate multiplier (proxy)
- Thrust vectoring via asymmetric exposure parameter (proxy)

Outputs
- Plots: thrust vs xi, mixing proxy vs xi, trade metric vs xi, vector angle vs asymmetry
- Tables printed to console at key xi values
"""

from __future__ import annotations
import math
import numpy as np
import matplotlib.pyplot as plt


# ---------- Core gas dynamics (quasi-1D, isentropic) ----------
def area_mach(M: float, gamma: float) -> float:
    """Isentropic area ratio A/A* as a function of Mach number M."""
    g = gamma
    term = (2.0 / (g + 1.0)) * (1.0 + (g - 1.0) * 0.5 * M * M)
    return (1.0 / M) * (term ** ((g + 1.0) / (2.0 * (g - 1.0))))


def mach_from_area(area_ratio: float, gamma: float, supersonic: bool = True) -> float:
    """
    Solve A/A*(M) = area_ratio for M using robust bisection.
    For rocket nozzles we normally want the supersonic branch at the exit.
    """
    if area_ratio <= 0:
        raise ValueError("area_ratio must be positive")

    lo, hi = (1.0001, 20.0) if supersonic else (1e-6, 0.9999)

    # Ensure bracket covers solution.
    # For supersonic branch, A/A* decreases near 1 then rises; but for M>1 it is monotonic increasing.
    # So bisection on [1+, hi] is safe as long as hi is large enough.
    for _ in range(140):
        mid = 0.5 * (lo + hi)
        fmid = area_mach(mid, gamma) - area_ratio
        flo = area_mach(lo, gamma) - area_ratio

        if np.sign(fmid) == np.sign(flo):
            lo = mid
        else:
            hi = mid

    return 0.5 * (lo + hi)


def isentropic_T(T0: float, M: float, gamma: float) -> float:
    """Static temperature from total temperature (calorically perfect)."""
    return T0 / (1.0 + (gamma - 1.0) * 0.5 * M * M)


def isentropic_p(p0: float, M: float, gamma: float) -> float:
    """Static pressure from total pressure (calorically perfect)."""
    return p0 / (1.0 + (gamma - 1.0) * 0.5 * M * M) ** (gamma / (gamma - 1.0))


def choked_mdot(P0: float, T0: float, At: float, gamma: float, R: float) -> float:
    """
    Choked mass flow rate for an ideal nozzle:
    mdot = P0*At*sqrt(T0) * sqrt(gamma/R) * (2/(gamma+1))^((gamma+1)/(2*(gamma-1)))
    """
    g = gamma
    return (P0 * At / math.sqrt(T0)) * math.sqrt(g / R) * (2.0 / (g + 1.0)) ** (
        (g + 1.0) / (2.0 * (g - 1.0))
    )


# ---------- Corrugation / lobes reduced-order closures ----------
def corrugation_models(
    xi: float,
    A_exit: float,
    r_exit_mean: float,
    M_i: float,
    M_e: float,
    p_i: float,
    rho_i: float,
    U_i: float,
    Ld: float,
) -> tuple[float, float]:
    """
    Phenomenological model for:
      - total-pressure loss across corrugated region (p0_ratio <= 1)
      - streamwise circulation proxy Gamma_raw (m^2/s), later calibrated

    xi in [0,1]: corrugation extent ratio Lc/Ld.
    """

    # Peak wall deflection scaling: representative ~5 deg at exit; scale with A_exit/r_exit_mean.
    delta_e_deg_ref = 5.0
    rel_amp_ref = 0.018 / 0.162  # representative geometry ratio
    rel_amp = A_exit / max(r_exit_mean, 1e-12)

    delta_e = math.radians(delta_e_deg_ref * (rel_amp / rel_amp_ref))

    # Total-pressure loss model:
    # Growth faster than linear with xi (cumulative shock + 3D boundary layer effects).
    # The coefficient is tuned to keep losses "plausible" in reduced-order sense (few % at mid xi).
    C_loss = 1.6
    M_avg = 0.5 * (M_i + M_e)
    dp0_over_p0 = C_loss * (xi**2) * (delta_e**2) * (M_avg**1.3)
    p0_ratio = 1.0 - dp0_over_p0
    p0_ratio = float(np.clip(p0_ratio, 0.65, 1.0))

    # Circulation proxy from circumferential pressure gradient:
    # Gamma_raw ~ (Δp/(rho*U)) * Lc, with Δp scaling with wall deflection and pressure level.
    Lc = xi * Ld
    delta_p = 0.28 * p_i * (delta_e / math.radians(5.0)) * (rel_amp / rel_amp_ref)
    Gamma_raw = 1.2 * (delta_p / (max(rho_i, 1e-12) * max(U_i, 1e-12))) * Lc

    return p0_ratio, Gamma_raw


def mixing_eff(Gamma: float, Ue: float, De: float) -> float:
    """
    Mixing efficiency proxy (0..1). Increases with vortex strength.
    This is a reduced-order surrogate, not a direct scalar dissipation or species mixing measure.
    """
    S = abs(Gamma) / (max(Ue, 1e-12) * max(De, 1e-12))
    return float(1.0 - math.exp(-6.0 * S))


def spread_mult(Gamma: float, Ue: float, De: float) -> float:
    """Jet spread-rate multiplier surrogate (>=1)."""
    S = abs(Gamma) / (max(Ue, 1e-12) * max(De, 1e-12))
    return float(1.0 + 3.5 * S)


# ---------- Thrust vectoring via asymmetric exposure ----------
def thrust_vector(T: float, asym: float, k_tv: float = 0.18) -> tuple[float, float, float]:
    """
    Proxy thrust-vectoring model for asymmetric corrugation exposure.
    asym in [0,1], alpha clipped to +/-10 deg.
    """
    alpha = np.clip(k_tv * asym, -math.radians(10.0), math.radians(10.0))
    Tx = T * math.cos(alpha)
    Ty = T * math.sin(alpha)
    return float(Tx), float(Ty), float(alpha)


# ---------- Stage performance wrapper ----------
def stage_perf(
    Pc: float,
    Tc: float,
    gamma: float,
    MW: float,
    At: float,
    Ae_ratio: float,
    Pa: float,
    xi: float,
    A_exit: float,
    r_exit_mean: float,
    Ld: float,
    Mi: float = 2.1,
) -> dict:
    """
    Compute thrust + mixing proxies for a given stage and corrugation extent xi.
    Returns a dict of performance metrics.
    """
    R = 8314.462618 / MW  # J/(kg*K), MW in kg/kmol

    # Baseline exit conditions from isentropic nozzle expansion
    Me = mach_from_area(Ae_ratio, gamma, supersonic=True)
    Te = isentropic_T(Tc, Me, gamma)
    pe = isentropic_p(Pc, Me, gamma)

    ae = math.sqrt(gamma * R * Te)
    Ve = Me * ae

    mdot = choked_mdot(Pc, Tc, At, gamma, R)

    Ae = At * Ae_ratio
    De = math.sqrt(4.0 * Ae / math.pi)

    # Conditions at corrugation initiation station (approx by isentropic state at Mi)
    Ti = isentropic_T(Tc, Mi, gamma)
    pi = isentropic_p(Pc, Mi, gamma)
    ai = math.sqrt(gamma * R * Ti)
    Ui = Mi * ai
    rhoi = pi / (R * Ti)

    # Corrugation effects (loss + circulation proxy)
    p0_ratio, Gamma_raw = corrugation_models(
        xi=xi,
        A_exit=A_exit,
        r_exit_mean=r_exit_mean,
        M_i=Mi,
        M_e=Me,
        p_i=pi,
        rho_i=rhoi,
        U_i=Ui,
        Ld=Ld,
    )

    # Calibrate circulation so representative xi=0.6 returns ~12 m^2/s (as used in the paper)
    p0r_ref, G_ref = corrugation_models(
        xi=0.6,
        A_exit=0.018,
        r_exit_mean=0.162,
        M_i=2.1,
        M_e=Me,
        p_i=pi,
        rho_i=rhoi,
        U_i=Ui,
        Ld=Ld,
    )
    scale = 12.0 / max(G_ref, 1e-12)
    Gamma = Gamma_raw * scale

    # Map total-pressure loss to reduced effective exit velocity
    Ve_eff = Ve * math.sqrt(p0_ratio)

    # Thrust (momentum + pressure)
    T = mdot * Ve_eff + (pe - Pa) * Ae

    return {
        "mdot": mdot,
        "Me": Me,
        "pe": pe,
        "Ve": Ve,
        "Ve_eff": Ve_eff,
        "T": T,
        "Ae": Ae,
        "De": De,
        "Gamma": Gamma,
        "eta_mix": mixing_eff(Gamma, Ve_eff, De),
        "spr": spread_mult(Gamma, Ve_eff, De),
        "p0_ratio": p0_ratio,
    }


# ---------- Main experiment runner ----------
def main() -> None:
    # Representative parameters used in the paper
    gamma = 1.24
    MW = 22.0
    At = math.pi * (0.060**2)  # throat radius 60 mm
    Ae_ratio = 7.3
    r_exit_mean = 0.162
    A_exit = 0.018
    Ld = 0.415

    stage1 = {"name": "Stage 1 (sea-level)", "Pc": 70e5, "Tc": 3500.0, "Pa": 101325.0}
    stage2 = {"name": "Stage 2 (vacuum)", "Pc": 40e5, "Tc": 3400.0, "Pa": 50.0}

    xis = np.linspace(0.0, 0.9, 37)

    def sweep(stage):
        T, mix, spr, p0 = [], [], [], []
        for xi in xis:
            r = stage_perf(
                Pc=stage["Pc"],
                Tc=stage["Tc"],
                gamma=gamma,
                MW=MW,
                At=At,
                Ae_ratio=Ae_ratio,
                Pa=stage["Pa"],
                xi=float(xi),
                A_exit=A_exit,
                r_exit_mean=r_exit_mean,
                Ld=Ld,
            )
            T.append(r["T"])
            mix.append(r["eta_mix"])
            spr.append(r["spr"])
            p0.append(r["p0_ratio"])
        return np.array(T), np.array(mix), np.array(spr), np.array(p0)

    T1, mix1, spr1, p01 = sweep(stage1)
    T2, mix2, spr2, p02 = sweep(stage2)

    # Illustrative trade metric: mixing benefit minus weighted loss penalty
    w = 3.5
    J1 = mix1 - w * (1.0 - p01)
    J2 = mix2 - w * (1.0 - p02)

    # Plots
    plt.figure()
    plt.plot(xis, T1 / 1000.0, label=stage1["name"])
    plt.plot(xis, T2 / 1000.0, label=stage2["name"])
    plt.xlabel("Corrugation extent ratio ξ")
    plt.ylabel("Thrust (kN)")
```

```
    plt.title("Thrust vs corrugation extent")
    plt.grid(True)
    plt.legend()
    plt.show()

    plt.figure()
    plt.plot(xis, mix1, label=stage1["name"])
    plt.plot(xis, mix2, label=stage2["name"])
    plt.xlabel("Corrugation extent ratio ξ")
    plt.ylabel("Mixing proxy (0–1)")
    plt.title("Mixing proxy vs corrugation extent")
    plt.grid(True)
    plt.legend()
    plt.show()

    plt.figure()
    plt.plot(xis, J1, label="Stage 1 trade metric")
    plt.plot(xis, J2, label="Stage 2 trade metric")
    plt.axvline(0.6, linestyle="--")
    plt.xlabel("Corrugation extent ratio ξ")
    plt.ylabel("J = mix_proxy - w*(1-p0_ratio)")
    plt.title("Trade metric (illustrative)")
    plt.grid(True)
    plt.legend()
    plt.show()

    print("Argmax xi (Stage1, Stage2):", float(xis[np.argmax(J1)]), float(xis[np.argmax(J2)]))

    # Tables
    key_xis = [0.0, 0.3, 0.6, 0.8, 0.9]
    for st in (stage1, stage2):
        print("\n", st["name"])
        print("xi   Thrust[kN]  mix_proxy  p0_ratio")
        for xi in key_xis:
            r = stage_perf(
                Pc=st["Pc"],
                Tc=st["Tc"],
                gamma=gamma,
                MW=MW,
                At=At,
                Ae_ratio=Ae_ratio,
                Pa=st["Pa"],
                xi=xi,
                A_exit=A_exit,
                r_exit_mean=r_exit_mean,
                Ld=Ld,
            )
            print(f"{xi:0.1f}  {r['T']/1000:9.2f}  {r['eta_mix']:0.3f}  {r['p0_ratio']:0.3f}")

    # Vectoring demo at xi=0.6 for Stage 1
    r_opt = stage_perf(
        Pc=stage1["Pc"],
        Tc=stage1["Tc"],
        gamma=gamma,
        MW=MW,
        At=At,
        Ae_ratio=Ae_ratio,
        Pa=stage1["Pa"],
        xi=0.6,
        A_exit=A_exit,
        r_exit_mean=r_exit_mean,
        Ld=Ld,
    )
    asym_levels = np.linspace(0.0, 1.0, 6)
    alpha_deg = []
    lateral_kN = []
    for a in asym_levels:
        Tx, Ty, alpha = thrust_vector(r_opt["T"], float(a))
        alpha_deg.append(math.degrees(alpha))
        lateral_kN.append(Ty / 1000.0)

    plt.figure()
    plt.plot(asym_levels, alpha_deg)
    plt.xlabel("Asymmetry command (0–1)")
    plt.ylabel("Thrust vector angle (deg)")
    plt.title("Vectoring authority at ξ=0.6 (reduced-order)")
    plt.grid(True)
    plt.show()

    print("\nVectoring at xi=0.6 (Stage 1):")
    for a, ad, ty in zip(asym_levels, alpha_deg, lateral_kN):
        print(f"asym={a:0.1f}  alpha={ad:5.2f} deg  lateral={ty:6.2f} kN")


if __name__ == "__main__":
    main()
```

Appendix 2

The following are the detailed results.

Thrust Performance (Figure 1): The thrust–corrugation-extent graph demonstrates that thrust decreases gradually as the corrugation extent ratio $\xi = L_c/L_d$ increases, reflecting the cumulative impact of three-dimensional flow turning, oblique shock interactions, and enhanced viscous dissipation introduced by the corrugated lobe geometry. As the corrugated section becomes longer, the effective total pressure at the nozzle exit is reduced, which in turn lowers the effective exhaust velocity and hence the momentum component of thrust. This reduction remains relatively modest over intermediate corrugation extents, indicating that moderate corrugation can be accommodated without severe performance penalties. The similarity in the trend for both sea-level and near-vacuum stages shows that the thrust sensitivity to corrugation length is primarily governed by internal compressible losses rather than ambient back pressure, underscoring that the principal efficiency cost of the corrugated architecture is intrinsic to the nozzle flow physics.

Mixing Enhancement (Figure 2): The mixing enhancement graph shows a monotonic increase in the mixing-efficiency proxy with increasing corrugation extent, illustrating how a longer corrugated region strengthens the generation and persistence of streamwise vortices within the exhaust plume. In the model, these vortices arise from circumferential pressure gradients imposed by the lobed geometry and their integrated strength grows with the axial length over which the corrugation is applied. As the non-dimensional vortex strength increases, the mixing proxy approaches saturation, representing accelerated entrainment and more rapid breakdown of the jet's potential core. The close correspondence between the curves for different stages indicates that the enhancement mechanism is largely geometric and non-dimensional in nature, and thus relatively insensitive to the absolute operating pressure or temperature of the stage.

Trade-Off Behavior (Figure 3): The trade-off graph combines the beneficial effect of enhanced mixing with the detrimental effect of total-pressure loss, producing a curve with a clear interior maximum. At small corrugation extents, the rapid rise in vortex-induced mixing outweighs the relatively small pressure losses, leading to an overall increase in the combined performance metric. Beyond a mid-range corrugation extent, however, additional increases in corrugation length produce diminishing returns in mixing while the loss mechanisms intensify, causing the combined metric to decline. This behavior highlights the existence of an optimal corrugation extent at which plume-conditioning benefits are maximized for an acceptable thrust penalty. The precise location of this optimum depends on how strongly mixing performance is weighted relative to thrust efficiency, but the presence of a peak itself demonstrates that partial corrugation is fundamentally more effective than either a smooth nozzle or a fully corrugated divergent section.

Thrust Vectoring Capability (Figure 4): The thrust vectoring graph illustrates how asymmetric exposure of the corrugated lobe geometry produces a controllable deflection of the thrust vector. As the asymmetry command increases, the thrust vector angle increases approximately linearly, reflecting the proportional relationship between geometric asymmetry and lateral momentum redistribution assumed in the reduced-order model. The saturation of the curve near the upper limit indicates a practical bound on vectoring authority, imposed to represent structural and flow-stability constraints. Physically, this result shows that once a sufficiently strong vortical structure is established by an intermediate corrugation extent, differential activation of the corrugated region can redirect the exhaust plume and generate meaningful lateral forces without mechanical nozzle gimbaling, providing a viable mechanism for non-mechanical thrust vector control in multi-stage propulsion systems.

Figure 1 (Thrust Performance):

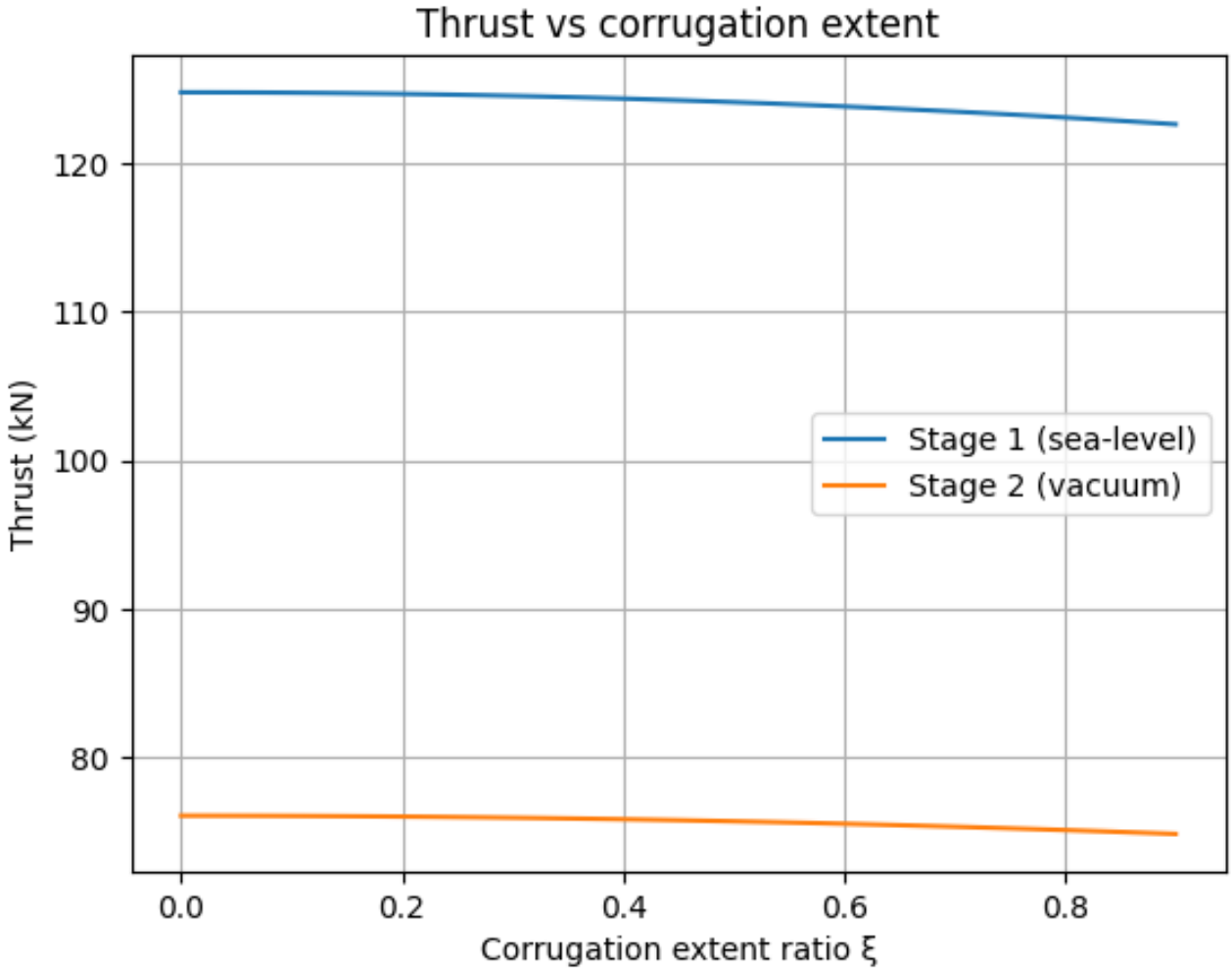

Figure 2 (Mixing Enhancement):

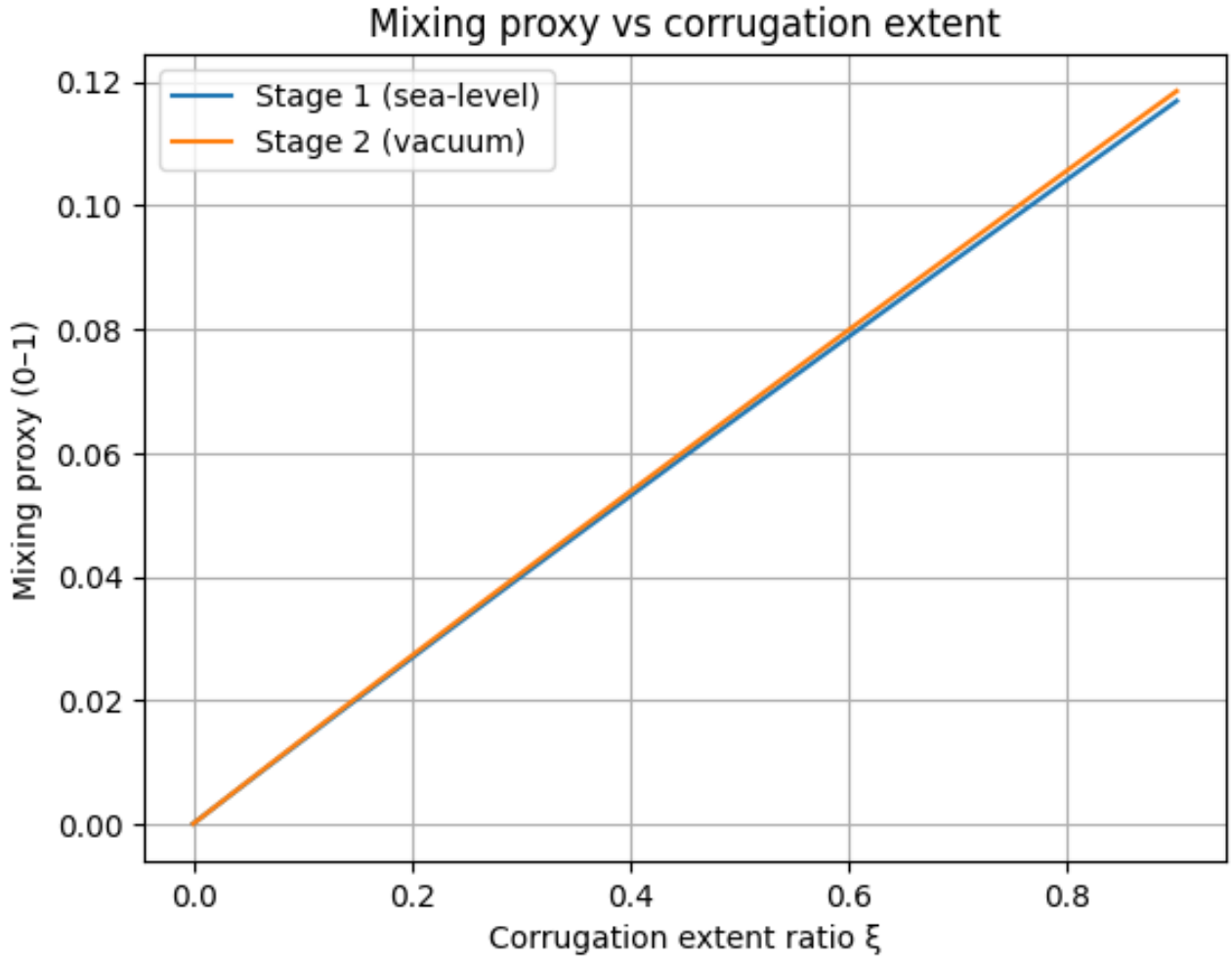

Figure 3 (Trade-Off Behavior):

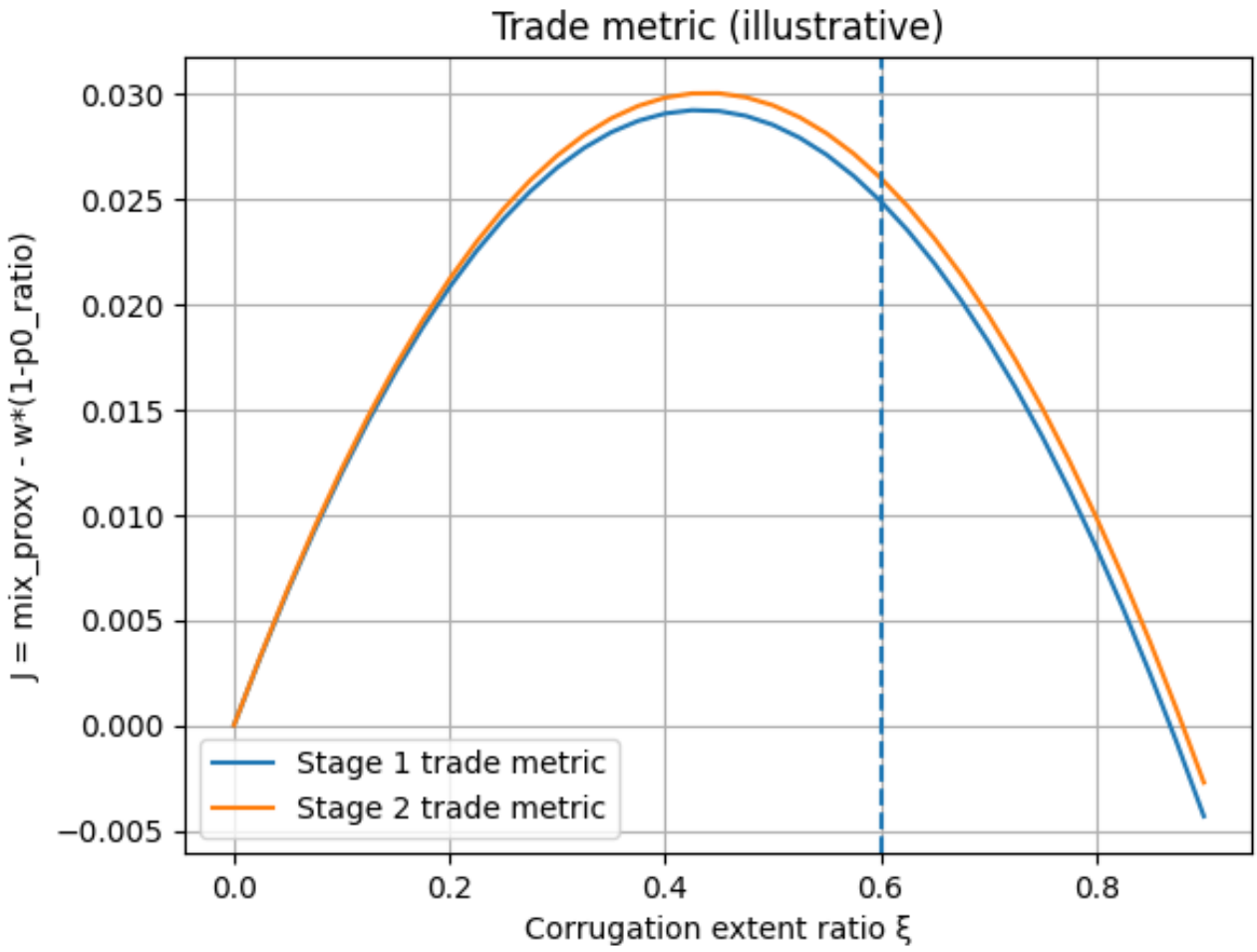

Figure 4 (Thrust Vectoring Capability):